A Fuel Consumption Optimization Model for the Multi-Period Inventory Routing Problem

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ABSTRACT

In traditional multi-period inventory routing problem (MIRP), traveling distance is considered as the only measurement of vehicles' variable transportation cost; however, in fact it is the cost of fuel consumption, not the distance, which is the greater concern. In this paper, we evaluate vehicles' variable transportation cost by fuel consumption, which is influenced by distance, load and fuel price. We present an integer program to formally characterize the fuel consumption considered MIRP (FCMIRP), which can help enterprises obtain more accurate tradeoff between transportation and inventory costs. It also benefits the environment, because reducing fuel consumption will curb carbon dioxide (CO_2) emissions. We add valid inequalities to strengthen the model and use a branch-and-cut algorithm. Computational tests indicate that FCMIRP can decrease fuel consumption and total cost over the traditional model. We also discuss factors that influence the results of FCMIRP.

Keywords: Multi-period inventory routing problem, fuel consumption, green logistics, valid inequalities, branch-and-cut algorithm

1. INTRODUCTION

With growing shortage of energy and its rising price, governments and enterprises have begun to pay close attention to sustainable development, in order to save energy and protect environment. For example, the US government has made energy policies to reduce fossil fuel consumption *(1)*. The United Nations and the European Union have enacted legislations to control energy consumption. Companies like Wal-Mart and IKEA have started to invest in renewable energy, hoping to decrease traditional energy (petroleum or coal) consumption.

Supply chain activities, such as production, transportation and inventory, all consume energy. Therefore, in recent years, researchers began to consider energy consumption in the supply chain *(2)*. Among all the logistics activities, transportation is regarded as one of the largest consumers of petroleum *(3)*. Thus, reducing fuel consumption in transportation will contribute to green logistics and cost reduction.

Generally, we divide transportation cost into two parts: (i) fixed cost, including driver wages, vehicle maintenance and depreciation, etc.; (ii) variable cost, which mainly refers to fuel cost *(4)*. Therefore, accurate computation of transportation cost can help plan more practical vehicle routes. Since fuel cost is related to many factors (distance, load, road condition, and speed, etc.) *(2, 4)*, it is not exact to only consider travel distance. For example, the fuel consumption of a fully loaded vehicle is always greater than an identical but empty vehicle when they are traveling along a specific route at the same speed.

Up to now, many studies on inventory routing problems (IRPs) only use distance as the sole measurement of variable transportation cost. As in IRPs we have to make a trade-off between transportation and inventory costs, if the computation of transportation cost is inaccurate, it will not only lead to suboptimal traveling schedules, but also fail to get exact inventory strategies. Therefore, in this paper we use fuel cost as vehicles' variable transportation cost. It is worth mentioning that considering fuel consumption in MIRP also benefits the environment, because transportation is the largest source of CO_2 emissions in logistics *(5)*, which are caused by fuel consumption.

To conclude, our study makes the following contributions:

- To the best of our knowledge, this paper is among the first to examine the influence of fuel cost on MIRP in detail.
- Numerical experiments demonstrate that FCMIRP can reduce overall cost and achieve a better environmental benefit. This could provide managerial insights for enterprises and governments in green logistics.

The remainder of the paper is organized as follows. Section 2 reviews related literature. Section 3 presents mathematical models for the FCMIRP and the traditional MIRP. Section 4 describes the solution method, and Section 5 presents numerical results. Conclusions follow in Section 6.

2. LITERATURE REVIEW

The Inventory Routing Problem (IRP) is a classic combinatorial optimization problem which determines simultaneously the optimal inventory strategy and vehicle schedules to minimize the supply chain's total cost. IRP has been extensively studied since its introduction in *(6)*. By considering different industrial applications and constraints, various types of IRPs have been formulated. In terms of industrial applications, IRPs is classified to road-based IRPs and maritime IRPs. As to constraints, IRPs can be defined according to following criteria: time horizon, supply chain topology, routing component, inventory policy, fleet composition and size *(7)*. For more details on IRPs see *(7, 8)*.

In MIRP, we minimize the total cost by determining retailers' optimal replenishment time and vehicle routes. *(9)* addresses a single period model which schedules multi-item replenishment with uncertain demand. *(10)* considers a many-to-one distribution network with multi-supplier and multi-product. *(11)* studies a multi-product multi-vehicle MIRP. There are also extensive studies on the algorithm for solving MIRP. The first type is exact algorithm, including the branch-and-cut (B&C) algorithm and the branch-price-and-cut (BPC) algorithm. *(12)* develops the first B&C algorithm. In their paper, only one vehicle is available for each period. *(13)* uses the B&C algorithm for several classes of MIRPs: multi-vehicle MIRPs with homogeneous and heterogeneous vehicles, MIRPs with transshipment options, and MIRPs with added consistency features. *(14)* uses a BPC algorithm to solve a one-to-many MIRP with maximum-level (ML) inventory replenishment policy. Under a ML policy, any quantity of products can be delivered to a retailer as long as the retailer's maximal inventory capacity is respected. The MIRP subsumes a VRP which is NP-hard, so (meta)heuristics are developed, including Variable Neighborhood Search *(15)*, Tabu Search *(16)* and Genetic Algorithm *(10)*, etc.

With increasing environmental pressures, researchers start to consider fuel consumption and CO_2 emissions in supply chain. *(17, 18)* summarize recent models for sustainable supply chain management (SSCM). In SSCM, one of the most extensively studied areas is green vehicle routing problem (VRP) which explicitly includes fuel or emission cost in VRP models. *(2)* addresses a capacitated VRP that minimizes fuel consumption. *(19)* explores cumulative VRP, in which both distance and vehicle load are considered for calculating fuel consumption. *(20)* compares different objective functions for VRP to quantify their influence on cost and emissions, including minimizing comprehensive cost, minimizing travel distance, minimizing weighted load, and minimizing energy consumption. *(21)* extends the traditional heterogeneous VRP objective to include fuel and emission costs, and analyzes their influences. *(22)* proposes to consider CO_2 emissions in facility location problem and suggests the difference between cost-minimization solutions and emission-minimization solutions. Detailed reviews about green logistics see *(1, 23)*.

3. PROBLEM FORMULATION

In this section, we present the mathematical models for the FCMIRP and the traditional MIRP.

3.1 Problem Description

We consider an outbound product distribution system consisting of a supplier and N geographically dispersed retailers. In each period, vehicles depart from the supplier to distribute products to retailers. Our assumptions include:

- The supplier only offers one type of products. At the beginning of each period, products are ready for distribution.
- Vehicle routes start and end at the supplier, and can be completed in one time period.
- Waiting, loading and unloading time at each vertex are not considered.
- Vehicles are homogeneous and capacitated.

• Split delivery is not allowed: a retailer is visited by at most one vehicle in each period.

• Retailers' demands are deterministic but variable over periods. Their demands must be fulfilled; hence backorders are not allowed.

• Retailers' inventory capacities are limited, and the ML replenishment policy is applied.

• Inventory holding costs are considered at both the supplier and the retailers. At the beginning of the time horizon, the initial inventory levels at each vertex are known.

The proposed FCMIRP uses the following notations:

Sets

$V = \{0, 1, \dots, N\}$ A set of all vertices, $i, j \in V$; vertex 0 is the supplier

$V' = \{1, 2, \dots N\}$ A set of retailers

$T = \{1, 2, \dots p\}$ Time horizon, $t \in T$

Parameters

d_i^t Retailer i's demand in period t (*kg*)

c_{ij} Distance between vertex i and vertex j (*km*)

Q The weight capacity of each vehicle (*kg*)

h_i Inventory holding cost at vertex i per unit weight per period (*$/kg/period*)

I_i^0 Initial inventory level of vertex i at the beginning of the time horizon (*kg*)

C_i Maximal inventory holding capacity at retailer i (*kg*)

r^t The product weight supplier receives (or produces) in period t (*kg*)

u Unit fuel price (*$/L*)

u' Variable transportation cost per unit distance per vehicle (*$/km*)

F Fixed vehicle cost per trip (*$/trip*)

f_d Driver wage per hour (*$/h*)

v Vehicle speed (*km/h*)

Decision variables

x_{ij}^t $x_{ij}^t = 1$ If arc (i, j) is traversed by a vehicle in period t, otherwise 0

a_{ij}^t The product weight carried by a vehicle through arc (i, j) in period t (*kg*)

q_i^t The product weight delivered to retailer i in period t (*kg*)

I_i^t The inventory level at vertex i at the end of period t (*kg*)

ρ_{ij}^t Fuel consumption rate of each vehicle from vertex i to vertex j in period t (*L/km*)

FC_{ij}^t Fuel cost of each vehicle from vertex i to vertex j in period t (*$*)

3.2 Fuel Cost

We use the model proposed by *(4, 5)* to compute fuel consumption. They state that fuel consumption mainly results from two elements: distance and fuel consumption rate (FCR)--fuel consumption per unit distance, where FCR is linearly associated with vehicle's load according to statistic data. Some other complicated models for fuel calculation are also available. For example, we can consider the effect of vehicle speed, road angle, acceleration, weather and traffic condition. However, here we choose to neglect some factors, based on following reasons: (i) it is found that in many researches the road angel and acceleration are set to 0, and that the speed is assumed to be constant *(21, 24, 25)*; (ii) factors such as weather and traffic are impractical and unreasonable to be quantified, because they vary significantly in different regions *(5)*; (iii) IRP is a problem that is even difficult than VRP, we have to keep the model tractable.

We first formulate FCR as

$$\rho_{ij}^t = \rho^0 + \frac{\rho^* - \rho^0}{Q} a_{ij}^t \tag{1}$$

where ρ^0, ρ^* are the empty-load and full-load fuel consumption rate respectively. As ρ^0, ρ^* and Q are fixed for a specific type of vehicles, we simplify equation (1) as:

$$\rho_{ij}^t = \rho^0 + \beta a_{ij}^t \text{, in which } \beta = \frac{\rho^* - \rho^0}{Q} \tag{2}$$

Therefore, the fuel cost from node i to node j in period t is

$$FC_{ij}^t = u\rho_{ij}^t c_{ij} = u\left(\rho^0 + \beta a_{ij}^t\right)c_{ij} \tag{3}$$

3.3 Mathematical Formulation

The completed formulation for FCMIRP is as follows:

$$\text{Min } f_1 = \sum_{t\in T}\sum_{i\in V} h_i I_i^t + \sum_{t\in T}\sum_{j\in V'} F x_{0j}^t + \sum_{t\in T}\sum_{i\in V}\sum_{j\in V} f_d c_{ij} x_{ij}^t / v + \sum_{t\in T}\sum_{i\in V}\sum_{j\in V} u(\rho^0 x_{ij}^t + \beta a_{ij}^t) c_{ij} \tag{4}$$

Subject to

$$I_0^t = I_0^{t-1} + r^t - \sum_{i\in V'} q_i^t \qquad \forall t \in T \tag{5}$$

$$I_i^t = I_i^{t-1} + q_i^t - d_i^t \qquad \forall i \in V', t \in T \tag{6}$$

$$I_i^t \geq 0 \qquad \forall i \in V, t \in T \tag{7}$$

$$I_i^t \leq C_i \qquad \forall i \in V', t \in T \tag{8}$$

$$q_i^t \leq C_i - I_i^{t-1} \qquad \forall i \in V', t \in T \tag{9}$$

$$a_{ij}^t \leq Q x_{ij}^t \qquad \forall i \in V, \forall j \in V,\ t \in T \tag{10}$$

$$\sum_{j\in V} x_{ij}^t = \sum_{j\in V} x_{ji}^t \qquad \forall i \in V, t \in T \tag{11}$$

$$\sum_{j\in V} a_{ij}^t = \sum_{j\in V} a_{ji}^t - q_i^t \qquad \forall i \in V', i \neq j, t \in T \tag{12}$$

$$\sum_{j\in V} x_{ji}^t \leq 1 \qquad \forall i \in V', t \in T \tag{13}$$

$$x_{ii}^t = 0 \qquad \forall i \in V, t \in T \tag{14}$$

$$a_{ij}^t \geq 0 \qquad \forall i \in V, j \in V, t \in T \tag{15}$$

$$q_i^t \geq 0 \qquad \forall i \in V', t \in T \tag{16}$$

$$x_{ij}^t \in \{0,1\} \qquad \forall i \in V, j \in V, t \in T \tag{17}$$

The objective function (4) minimizes the total cost, including inventory cost, fleet cost, driver wage, and fuel cost. Constraints (5) and (6) are the inventory balance equations at the supplier and the retailers respectively. Constraints (7) mean that inventories cannot be negative at each vertex. Constraints (8) impose maximal inventory level at retailers. Constraints (9) limit the product weight delivered to each retailer, to satisfy demand and to respect maximal inventory capacity. Constraints (10) guarantee that vehicles' capacities are not violated. Constraints (11) indicate that the number of vehicles leaving a node equal to that arriving at a node. Constraints (12) are the product flow balance equations at retailers and eliminate all subtours. Constraints (13) ensure that split delivery is not allowed. Constraints (14)-(17) define variable types.

We also give the mathematical formulation for traditional MIRPs to make a contrast:

$$\text{Min } f_2 = \sum_{i\in V}\sum_{t\in T} h_i I_i^t + \sum_{t\in T}\sum_{j\in V'} F x_{0j}^t + \sum_{t\in T}\sum_{i\in V}\sum_{j\in V} f_d c_{ij} x_{ij}^t / v + \sum_{i\in V}\sum_{j\in V}\sum_{t\in T} u' c_{ij}\, x_{ij}^t \tag{18}$$

Subject to (5)-(17) and

$$\sum_{i\in V'} q_i^t = \sum_{j\in V'} a_{0j}^t \qquad \forall t \in T \tag{19}$$

Constraints (19) ensure that in each period the product weight delivered to all retailers equals to that provided by the supplier. If this constraint is absented from the traditional model, sometimes a vehicle may carry more products (more than the weight delivered to all retailers) from the supplier, then at the end of the trip the vehicle carries surplus products back to the supplier. However, these constraints are not necessary in FCMIRP, otherwise vehicles will consume more fuel, which is not a wise choice for FCMIRP.

4. VALID INEQUALITY AND SOLUTION METHOD

4.1 Valid Inequalities

(11, 12, 26) have introduced several classes of valid inequalities for the IRPs. We extend some of them to strengthen our models.

$$\sum_{j\in V}\sum_{l=1}^{t} x_{ji}^l \geq \left\lceil \left(\sum_{l=1}^{t-1} d_i^l - I_i^0\right) / C_i \right\rceil \qquad \forall i \in V', \forall t \in T \tag{20}$$

Inequalities (20) mean that for retailer i, if its total demand over $[1, t-1]$ is greater than its initial inventory level, then it has to be visited at least the number of times that corresponds to the right-hand side of the inequality in the interval $[1, t]$.

We extend constraints (20) to any time interval $[t_1, t_2]$ and consider that the maximal delivery weight is the minimum between vehicle's capacity and retailers' inventory capacities. This yields inequalities (21):

$$\sum_{j\in V}\sum_{t'=t_1}^{t_2} x_{ji}^{t'} \geq (\sum_{t'=t_1}^{t_2} d_i^{t'} - I_i^{t_1-1})/\min\{Q, C_i\} \qquad \forall i \in V', \forall t_1, t_2 \in T, t_2 \geq t_1 \tag{21}$$

For retailer i, if its inventory at the end of period t_1-1 is sufficient to meet its demands over $[t_1, t_2]$, then it is not mandatory for vehicles to visit it. Otherwise, there has to be at least one visit to retailer i during $[t_1, t_2]$. This yields inequalities (22):

$$\sum_{j\in V}\sum_{t'=t_1}^{t_2} x_{ji}^{t'} \geq (\sum_{t'=t_1}^{t_2} d_i^{t'} - I_i^{t_1-1})/\sum_{t'=t_1}^{t_2} d_i^{t'} \qquad \forall i \in V', \forall t_1, t_2 \in T, t_2 \geq t_1 \tag{22}$$

Note that the right-hand side of equation (20) is rounded up, because it takes a constant value. However, this is not allowed in equations (21) and (22), because their numerators contain variables, which will make them nonlinear otherwise.

4.2 Solution Method

To exactly display the different solutions obtained from the two models, exact algorithms are preferred in this study. Therefore, we use a B&C algorithm here. We first add valid inequalities (20)-(22) into the model at the root node of the search tree. Then we use a commercial solver to solve the linear relaxation problems at each node. The best bound strategy is employed to select nodes.

5. COMPUTATIONAL EXPERIMENTS AND ANALYSES

The aim of this section includes:

- To display the solution differences between the two models in detail;
- To discuss the factors that influence FCMIRP's decisions.

5.1 Instance Data

As to MIRP, we use data sets generated by *(11)* for a single product. The names of instances indicate the numbers of retailers, product types, vehicles, periods, and the instance number. For example, label "20-1-3-5-4" is the fourth instance for 20 customers, 1 type of products, 3 vehicles and 5 periods. Note that the number of vehicles is unlimited in our study. For a specific combination of customer and period, there are 15 instances by changing the number of vehicles. The distance between nodes is computed as $c_{ij} = \lfloor \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + 0.5/\alpha} \rfloor$, where ($x_i$,$y_i$) are the coordinates of vertex i, and α=6. The vehicle speed $v = 80$. As to fuel consumption, according to *(27)*, we set $\rho^0 = 0.296$ and $\rho^* = 0.390$. Based on (*21, 24, 25*), we set $F = 35.19$, $f_d = 10.35$, u =0.96, $u' = 0.75$.

The branch-and-cut algorithm was implemented in Python language using Gurobi 7.5.1 as the solver. 1 thread is used. All computations were executed on a PC with an Intel Core i5 Processor (2.3GHz) and 8G memory. A time limit of 1 hour was imposed on each instance.

5.2 Comparisons between Models

5.2.1 Cost Configuration

To study the differences between models, we present the comparison in four aspects: fuel consumption, inventory cost, transportation cost and total cost. Only instances with optimal solutions under the time limit are reported. Note that in traditional MIRP, the amount of fuel consumption might be different for a given route, because the distance matrix is symmetric and a vehicle may travel from two opposite directions. In following analyses, the worst-case fuel consumption is reported for the traditional model to indicate the maximal economic and environmental benefits we can achieve from FCMIRP. Detailed comparison results are given in Table 1 and graphical comparison is presented in Figure 1 and Figure 2. For ease of exposition, we give each instance a new number, which can refer to Table 1.

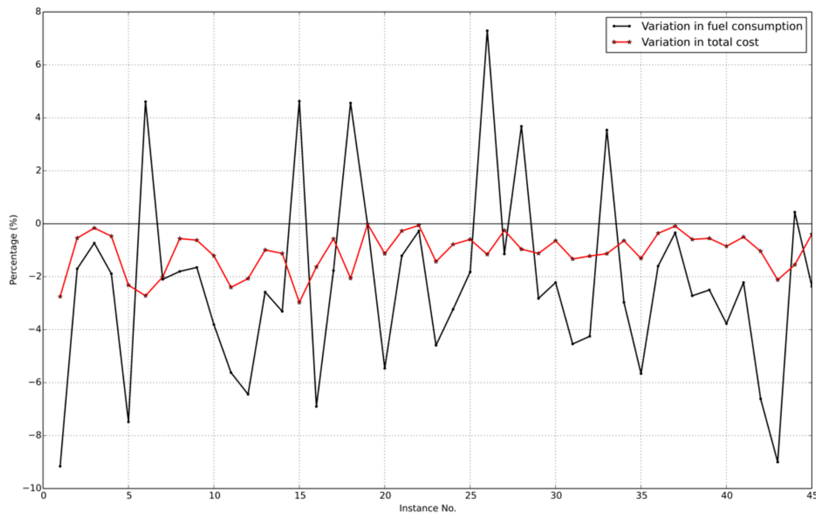


FIGURE 1 Gap in fuel consumption and total cost compared with traditional MIRP

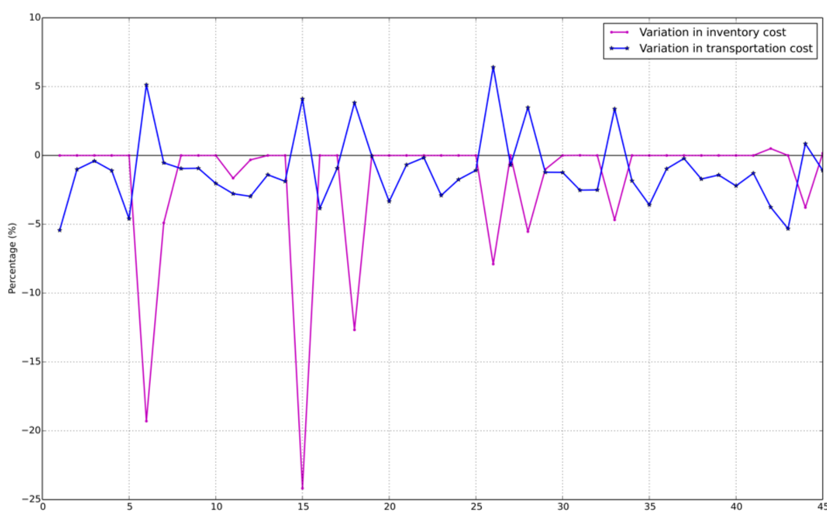


FIGURE 2 Gap in inventory and transportation cost compared with traditional MIRP

From Table 1 and Figure 1, in most instances (38 out of 45, about 84.4%), the fuel consumption of the FCMIRP is less, and it can achieve a saving of 2.21% on average. For instances 1, 5, 12, 16, 42 and 43, the saving is over 6%. Since fuel consumption is the direct cause of CO_2 emissions, we conclude that in most cases, FCMIRP can help enterprises achieve a better environmental benefit. Moreover, in all instances, FCMIRP generates solutions with lower cost, leading to an average saving of 1.09%. Figure 2 shows that in some cases, even though the inventory cost of the two models is the same, the transportation cost of the FCMIRP is lower, because less fuel is consumed. For instances 6, 15, 18, 26, 28, and 33, the transportation cost has a slight increase, in order to lower inventory cost. This suggests that considering fuel consumption in MIRP will lead to a new balance between inventory and transportation costs.

In 7 instances, vehicles consume more fuel in the FCMIRP. This counter-intuitive phenomenon can be explained by the fact that the FCMIRP is cost-oriented, and sometimes a small increase in fuel consumption can lead to a large decrease in inventory cost, result in lower total cost. In following sections, we will use four instances to exemplify the specific differences in solutions of these two models.

TABLE 1 Detailed Comparison between the FCMIRP and the Traditional MIRP

		FCMIRP				Traditional MIRP				Gap (%)			
NO.	Instance	Fuel (*L*)	Inventory cost (*$*)	Transport cost (*$*)	Total cost (*$*)	Fuel (*L*)	Inventory cost (*$*)	Transport cost (*$*)	Total cost (*$*)	Fuel	Inventory cost	Transport cost	Total cost
1	10-1-1-3-1	142.94	225.88	231.66	457.54	156.03	225.88	244.23	470.11	-9.16	0.00	-5.43	-2.75
2	10-1-1-3-2	146.91	208.85	238.71	447.56	149.41	208.85	241.11	449.96	-1.70	0.00	-1.01	-0.54
3	10-1-1-3-3	92.64	239.77	162.80	402.57	93.31	239.77	163.45	403.22	-0.73	0.00	-0.40	-0.16
4	10-1-1-3-4	127.75	281.30	211.13	492.43	130.17	281.30	213.45	494.75	-1.89	0.00	-1.10	-0.47
5	10-1-1-3-5	179.55	273.79	281.04	554.83	192.98	273.79	293.93	567.72	-7.48	0.00	-4.59	-2.32
6	10-1-3-3-1	228.82	180.61	382.04	562.64	218.27	215.47	362.46	577.93	4.61	-19.30	5.13	-2.72
7	10-1-3-3-2	199.64	174.04	340.3	514.34	203.82	182.56	342.12	524.68	-2.10	-4.90	-0.53	-2.01
8	10-1-3-3-3	178.05	220.08	311.3	531.38	181.26	220.08	314.25	534.33	-1.80	0.00	-0.95	-0.56
9	10-1-3-3-4	200.75	168.97	341.5	510.47	204.06	168.97	344.68	513.65	-1.65	0.00	-0.93	-0.62
10	10-1-3-3-5	159.30	193.41	286.57	479.99	165.36	193.41	292.39	485.80	-3.81	0.00	-2.03	-1.21
11	10-1-5-3-1	214.20	203.92	396.33	600.24	226.23	207.28	407.36	614.64	-5.62	-1.65	-2.78	-2.40
12	10-1-5-3-2	194.80	190.79	370.98	561.77	207.34	191.41	381.99	573.40	-6.44	-0.32	-2.97	-2.07
13	10-1-5-3-3	253.05	189.79	446.57	636.35	259.58	189.79	452.84	642.63	-2.58	0.00	-1.40	-0.99
14	10-1-5-3-4	207.69	236.50	350.62	587.12	214.56	236.50	357.22	593.72	-3.31	0.00	-1.88	-1.12
15	10-1-5-3-5	290.00	165.49	495.75	661.24	276.56	205.51	475.35	680.86	4.63	-24.18	4.11	-2.97
16	10-1-1-5-3	206.90	479.58	353.74	833.32	221.17	479.58	367.31	846.89	-6.90	0.00	-3.84	-1.63
17	10-1-3-5-2	350.40	380.92	611.56	992.48	356.59	380.92	617.25	998.17	-1.77	0.00	-0.93	-0.57
18	10-1-3-5-5	351.23	360.42	647.94	1008.35	335.20	406.07	623.1	1029.17	4.56	-12.67	3.83	-2.06
19	20-1-1-3-1	194.32	522.88	301.82	824.71	194.44	522.88	301.93	824.81	-0.06	0.00	-0.04	-0.01
20	20-1-1-3-2	179.67	552.59	282.32	834.91	189.48	552.59	291.74	844.33	-5.46	0.00	-3.34	-1.13
21	20-1-1-3-3	160.55	386.60	254.4	641.00	162.49	386.60	256.13	642.73	-1.21	0.00	-0.68	-0.27
22	20-1-1-3-4	171.38	447.59	270.09	717.68	171.83	447.59	270.53	718.12	-0.27	0.00	-0.16	-0.06
23	20-1-1-3-5	230.76	360.69	351.16	711.86	241.35	360.69	361.33	722.02	-4.59	0.00	-2.90	-1.43
24	20-1-3-3-1	206.56	440.33	350.96	791.29	213.24	440.33	357.11	797.44	-3.23	0.00	-1.75	-0.78
25	20-1-3-3-2	271.27	361.31	438.05	799.37	276.20	361.31	442.78	804.09	-1.82	0.00	-1.08	-0.59

Table 1 continues

NO.	Instance	FCMIRP Fuel (*L*)	FCMIRP Inventory cost (*$*)	FCMIRP Transport cost (*$*)	FCMIRP Total cost (*$*)	Traditional MIRP Fuel (*L*)	Traditional MIRP Inventory cost (*$*)	Traditional MIRP Transport cost (*$*)	Traditional MIRP Total cost (*$*)	Gap (%) Fuel	Gap (%) Inventory cost	Gap (%) Transport cost	Gap (%) Total cost
26	20-1-3-3-3	225.03	418.64	372.96	791.60	208.63	451.69	349.06	800.75	7.29	-7.89	6.41	-1.16
27	20-1-3-3-4	173.44	481.09	268.32	749.41	175.41	481.09	270.21	751.30	-1.14	0.00	-0.70	-0.25
28	20-1-3-3-5	221.94	360.41	370.38	730.79	213.76	380.35	357.48	737.83	3.68	-5.53	3.48	-0.96
29	20-1-5-3-1	276.93	411.56	478.67	890.23	284.75	415.70	484.5	900.20	-2.82	-1.01	-1.22	-1.12
30	20-1-5-3-2	277.29	448.62	479.67	928.28	283.45	448.62	485.58	934.20	-2.22	0.00	-1.23	-0.64
31	20-1-5-3-3	275.73	423.72	477.52	901.25	288.26	423.66	489.55	913.21	-4.54	0.01	-2.52	-1.33
32	20-1-5-3-4	248.58	428.00	406.31	834.31	259.15	428.00	416.45	844.45	-4.25	0.00	-2.50	-1.22
33	20-1-1-5-2	344.39	684.85	537.48	1222.33	332.19	716.87	519.3	1236.17	3.54	-4.68	3.38	-1.13
34	30-1-1-3-1	206.07	598.85	319.31	918.16	212.20	598.85	325.19	924.04	-2.97	0.00	-1.84	-0.64
35	30-1-1-3-2	240.21	633.86	363.34	997.20	253.80	633.86	376.39	1010.25	-5.66	0.00	-3.59	-1.31
36	30-1-1-3-3	224.93	571.96	342.46	914.42	228.52	571.96	345.78	917.74	-1.60	0.00	-0.97	-0.36
37	30-1-1-3-4	247.80	587.98	375.16	963.14	248.66	587.98	375.98	963.96	-0.34	0.00	-0.22	-0.09
38	30-1-1-3-5	218.64	626.66	333.31	959.98	224.57	626.66	339.01	965.67	-2.72	0.00	-1.71	-0.59
39	30-1-3-3-1	218.19	587.88	367.94	955.83	223.64	587.88	373.17	961.05	-2.50	0.00	-1.42	-0.55
40	30-1-3-3-2	247.86	651.09	404.84	1055.93	257.20	651.09	413.8	1064.89	-3.77	0.00	-2.21	-0.85
41	30-1-3-3-3	238.06	616.95	392.85	1009.8	243.35	616.95	397.93	1014.88	-2.22	0.00	-1.29	-0.50
42	30-1-3-3-4	222.05	658.25	371.91	1030.17	236.74	654.99	385.88	1040.87	-6.61	0.50	-3.76	-1.04
43	30-1-3-3-5	264.42	649.00	429.53	1078.53	288.22	649.00	452.38	1101.38	-9.00	0.00	-5.32	-2.12
44	30-1-5-3-1	323.47	580.91	540.69	1121.59	322.03	602.86	536.07	1138.93	0.44	-3.78	0.85	-1.55
45	30-1-5-3-2	271.55	562.45	436.51	998.96	277.99	561.55	441.27	1002.82	-2.37	0.16	-1.09	-0.39
	Average	224.57	409.53	372.86	782.39	228.97	414.86	375.80	790.66	-2.21	-1.89	-1.00	-1.09

5.2.2 Route Direction and Order

FCMIRP will lead to a difference in route direction, because decision makers will only choose the direction in which vehicles consume less energy. However, in traditional MIRP, a vehicle may travel in either direction. Take instance "10-1-1-3-1" for example, in the two models the route length and inventory cost are the same while the route direction is converse, shown in Table 2.

From Table 2 one may conclude that we can directly transform traditional MIRP to FCMIRP by simply reversing the route direction, but this method does not always work. Take instance "10-1-3-5-2" for example, presented in Table 3. For traditional MIRP, if we reverse one route in period 2, which will become "0→6→9→2→0", then the fuel consumption will be 47.64. However, if we further exchange the visiting order of retailers 2 and 9, just as the route in the FCMIRP, the fuel consumption will be 45.27. Therefore, a simple reverse operation cannot guarantee a maximal saving of energy.

TABLE 2 Instance 10-1-1-3-1 to Show the Difference in Route Direction

FCMIRP	Traditional MIRP
Period 1: No route	Period 1: No route
Period 2: 0→9→4→5→6→2→3→1→10→0	Period 2: 0→10→1→3→2→6→5→4→9→0
(fuel consumption: 142.94)	(fuel consumption: 156.03)
Period 3: No route	Period 3: No route

TABLE 3 Instance 10-1-3-5-2 to Show the Difference in Route Order

FCMIRP		Traditional MIRP	
Periods 1, 2: No route		Periods 1, 2: No route	
Period 3: 0→2→3→4→0	0→7→10→8→1→0	Period 3: 0→4→3→2→0	0→1→8→10→7→0
Period 4: 0→6→**2**→**9**→0	0→1→5→8→0	Period 4: 0→**2**→**9**→6→0	0→8→5→1→0
Period 5: No route		Period 5: No route	

5.2.3 Inventory Strategy

Sometimes even though the routes of the two models are similar, their inventory strategy may be different. Take instance "10-1-5-3-1" for example.

TABLE 4 Instance 10-1-5-3-1 to Show the Difference in Inventory Strategy

FCMIRP	Traditional MIRP
Periods 1: No route	Period 1: No route
Period 2: 0→10(45)[a]→4(116)→9(37)→5(28)→1(1)→0	Period 2: 0→10(45)→4(116)→9(37)→5(28)→1(1)→0
0→2(61)→8(119)→0	0→2(61)→8(119)→**7(24)**→0
Period 3: 0→2(75)→**7(24)**→0	Period 3: 0→2(75)→0

10(45)[a] means retailer 10 is visited and the product weight delivered is 45 kg.

Table 4 shows that the routes of the two models are very similar except that retailer 7 is visited in different periods. For traditional MIRP, visiting retailer 7 in period 2 will reduce route length, result in lower driver cost and variable transportation cost. However, for FCMIRP, it is better to visit retailer

7 later, because its inventory holding cost coefficient is much higher than that of the supplier, and the saving of inventory cost will exceed the increase of driver cost.

5.2.4 Route Schedule and Inventory Strategy

FCMIRP may result in totally different route schedule and inventory strategy. Take instance "10-1-5-3-2" as an example, shown in Table 5. It demonstrates that the number of trips in each period is different and that the total product weight delivered to a retailer (like retailer 10) varies. From instance data, we find that the inventory holding cost coefficient of retailer 10 is 0, then in traditional MIRP more products are carried to the retailer, in order to control inventory cost, which on the other hand causes more fuel consumption.

TABLE 5 Instance 10-1-5-3-2 to Show the Difference in Route Schedule and Inventory Strategy

FCMIRP	Traditional MIRP
Periods 1: No route	Period 1: No route
Period 2: 0→**1(43)**→3(71)→4(88)→9(17)→0	Period 2: 0→**10(44)** →9(17)→4(88)→3(71)→0
Period 3: 0→**1(86)**→6(52)→7(64)→0	0→**1(53)**→0
0→2(28)→**10(10)**→0	Period 3: 0→2(28)→7(64)→6(52)→**1(76)**→0

To conclude, the solution of the two models differs from each other in terms of route direction, route order, and inventory strategy, which leads to a new tradeoff between inventory and transportation cost, and also generates different environmental implications.

5.3 Parameter Analyses

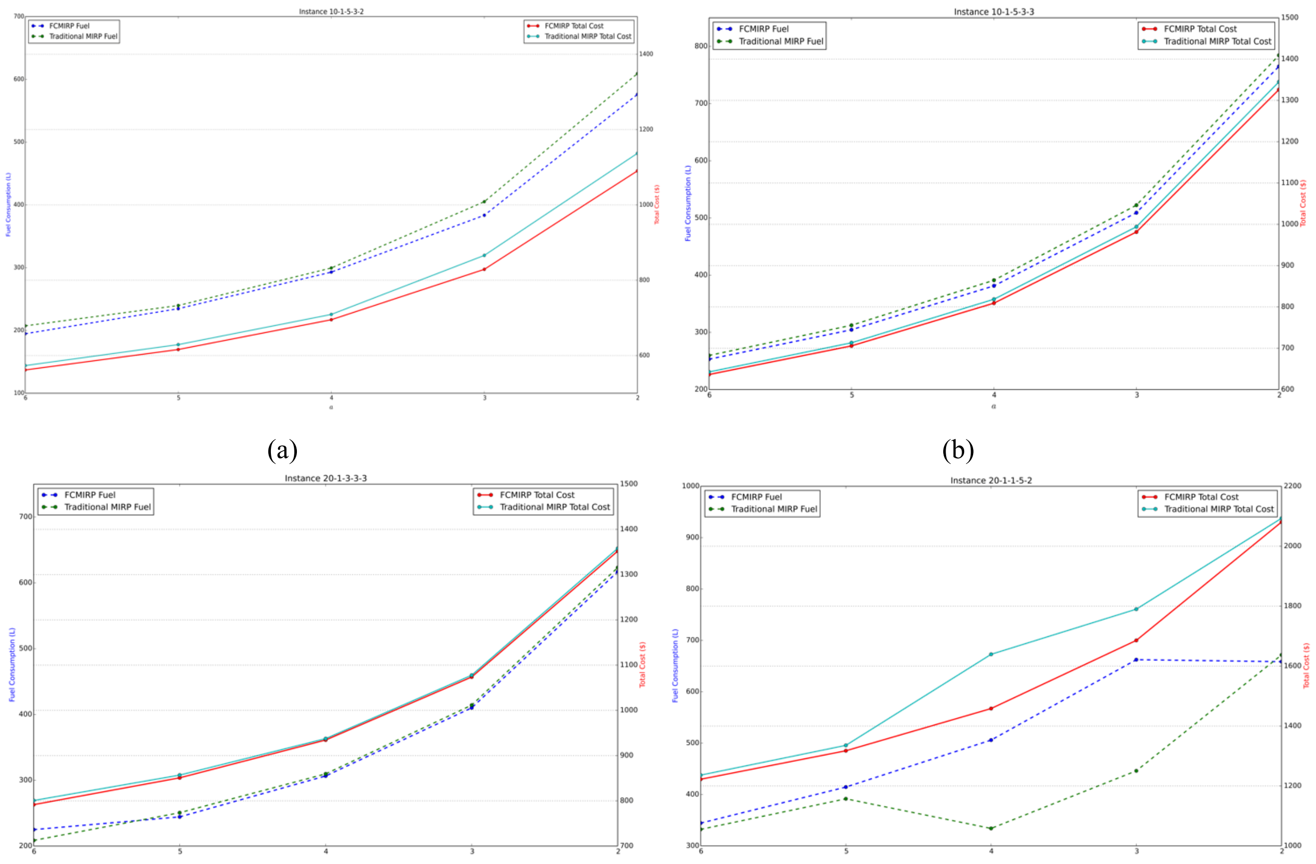
We now examine the factors that may influence the decision of FCMIRP. We perform our analyses base on randomly selected 2 small instances (10-1-5-3-2, 10-1-5-3-3) and 2 medium instances (20-1-3-3-3, 20-1-1-5-2), because they are more time-efficient compared with large ones.

5.3.1 Distance Factor

Since we have scaled down the distance between nodes in previous test, to consider the situation that during each trip driver's work time is shorter than 8 hours (actually our results show that the average work time is 5.12 hours per trip). Here we relax this constraint to model the cases of long distance transport, in order to figure out FCMIRP's potential for this application. We gradually increase the distance between nodes by changing α from 6 to 2 and observe its effect (shown in Figure 3), other parameters' values are the same as in Section 5.1.

It shows that as distance increases the total cost will increase as expected in these two models. However, the total cost of the FCMIRP is lower under any case. The average saving for each instance is 2.99%, 1.18%, 0.60%, and 4.34%, respectively. The maximal saving is 12.41% for instance 20-1-1-5-2 with α=4. Thus, we conclude that the FCMIRP is also capable of saving cost for long distance transport, and sometimes it has a great potential to reduce cost.

In terms of fuel consumption, the general trend is increasing for both models. For the first two instances, the FCMIRP can save 4.45% and 2.57% fuel on average. For the last instance, FCMIRP consume more fuel in order to lower inventory cost and total cost.



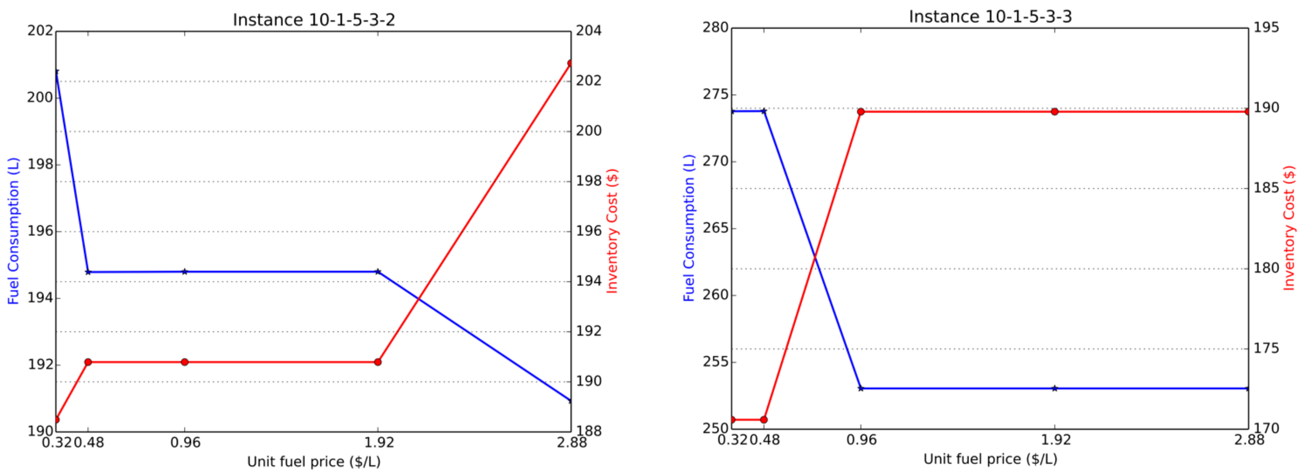
(a) (b)

(c) (d)

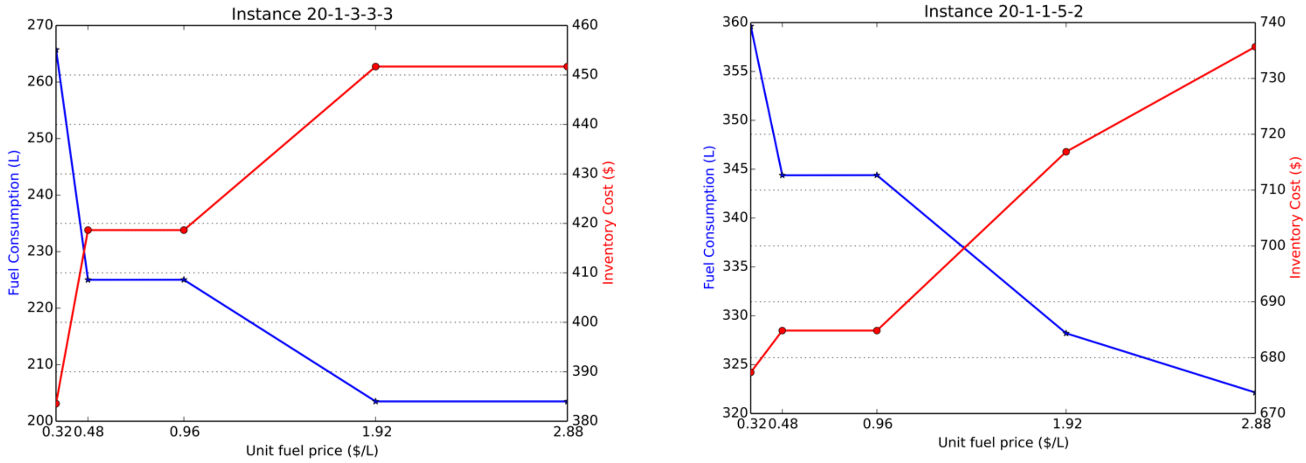
FIGURE 3 The influence of distance factor

5.3.2 Unit Fuel Price

For FCMIRP, we change u step by step and observe its influence on fuel consumption and inventory cost.



(a) (b)



(c) (d)

FIGURE 4 Trend of inventory cost and fuel consumption as u changes

As expected, when u increases the general trend of inventory cost is increasing and that of fuel consumption is decreasing. However, we find that sometimes when the fuel price varies, both the inventory cost and fuel consumption keep constant. In these situations, if decision makers choose to further increase inventory expenses, it will cost more money, more than the decrease of transportation cost. Therefore, it is better to keep current decisions. Since CO_2 emissions and fuel consumption is linearly dependent, we can also recognize the fuel price here as emission price (or carbon taxing). Then Figure 4 can also represent the trend of CO_2 emissions with varying carbon taxing. Thus, we conclude that higher fuel or emission price does not always mean a lower emission level.

5.3.3 Fleet Size

We use instances 10-1-5-3-2 and 20-1-3-3-3 to analyze the influence of fleet size, shown in Table 6. We add constraint (23) to both models, where s is the maximal allowable trip number in each period. Meanwhile, we halve the vehicle capacity in each instance.

$$\sum_{j\in V'} x_{0j}^{t} \leq s \qquad\qquad t \in T \tag{23}$$

TABLE 6 The Influence of Fleet Size

	10-5-3-2							20-3-3-3					
	FCMIRP		Traditional MIRP		Gap (%)			FCMIRP		Traditional MIRP		Gap (%)	
s	Fuel	Cost	Fuel	Cost	Fuel	Cost	*s*	Fuel	Cost	Fuel	Cost	Fuel	Cost
2	282.13	770.68	291.51	780.43	-3.32	-1.27	1	270.55	928.63	283.33	942.81	-4.72	-1.53
3	282.13	767.32	291.51	777.07	-3.32	-1.27	2	270.32	897.66	281.19	908.53	-4.02	-1.21

Table 6 shows that FCMIRP can still save fuel and total cost when this new constraint is added. As expected, when the allowable trip number is small, the total cost will increase, which mainly results from increasing inventory cost. This is because if fewer vehicles are available, we have to move some trips to the next a few periods to be executed, and the inventory holding cost coefficient of the supplier is higher than that of some retailers.

6. CONCLUSION

In this paper, we address a MIRP with environmental consideration. Being different from previous studies, a new model considering fuel consumption is proposed, which can help enterprises make more accurate tradeoff between transportation and inventory costs, and assist them to lower fuel consumption and CO_2 emissions. Numerical tests demonstrate that FCMIRP indeed generates different vehicle schedules and inventory strategies. For all instances, the FCMIRP is capable of saving overall cost, and in most cases it can decrease fuel consumption and CO_2 emissions.

Based on the numerical tests, we can derive managerial insights from two sides: (1) enterprises should reconsider their inventory replenishment (tactic) plan and vehicle routing (operational) plan by taking fuel consumption into account, which are proved to be quite different from the traditional distance-based plans; and (2) for regulators of the government, higher fuel price or carbon taxing do not always lead to a better environmental benefit, since to pursue overall profits, enterprises may sacrifice the transportation costs (therefore more emissions) for the saving of inventory costs in case that inventory costs are major components.

Future studies could be conducted in two aspects. First, we can develop powerful heuristic algorithms to solve large FCMIRP instances. Second, more factors can be incorporated when calculating fuel consumption, such as vehicle speed, road angles, and different vehicle type.

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