A Two-Stage Robust Approach for the Reliable Logistics Network Design Problem

Chun Cheng[a,b,c], Mingyao Qi[c,*], Ying Zhang[d], Louis-Martin Rousseau[a,b]

[a] Department of Mathematics and Industrial Engineering, Polytechnique Montréal, Montréal H3C 3A7, Canada

[b] CIRRELT, Montréal H3C 3A7, Canada

[c] Research Center on Modern Logistics, Graduate School at Shenzhen, Tsinghua University, Shenzhen 518055, China

[d] Zhejiang Cainiao Supply Chain Management Co., Ltd., Hangzhou 310000, China

Abstract

This paper examines a three-echelon logistics network in which all supply and transshipment nodes are subject to disruption. We use uncertainty sets to describe the possible scenarios without depending on probabilistic information. We adopt a two-stage robust optimization approach where location decisions are made before and recourse decisions are made after the disruptions are known. We construct three two-stage robust models, which are solved exactly by a column-and-constraint-generation algorithm. Numerical tests demonstrate that the proposed algorithm outperforms the Benders decomposition method in both solution quality and computational time, and that the system's reliability can be improved with only a slight increase in the normal cost.

Keywords: Reliable logistics network design; facility disruption; two-stage robust optimization; column-and-constraint-generation algorithm

1 Introduction

The logistics network design problem (LNDP) is key to achieving efficient operations among suppliers, manufacturers, and customers (Min and Zhou 2002). Compared to the classical facility location problem, it considers multiple echelons and decides the number of suppliers and warehouses, their locations and capacities, and the product flow throughout the network (Pishvaee et al. 2010). The LNDP decisions are strategic: once facilities are built, they are expected to run long term, because it is normally expensive to open and close facilities. Tactical (e.g., supplier and distribution channel selections) and operational (e.g., vehicle scheduling) decisions are based on strategic decisions. Therefore, the value of logistics network design is acknowledged in both academia and industry (Cordeau et al. 2006, Peng et al. 2011, Melo et al. 2009).

One importance aspect in LNDP is to deal with uncertainty, like uncertain set-up costs of facilities, uncertain transportation costs and customer demands (Alumur et al. 2012, Mišković et al. 2017).

Corresponding author. E-mail address: qimy@sz.tsinghua.edu.cn

Facility disruption is another type of uncertainty. Natural disasters (earthquakes, hurricanes, etc.) or industrial accidents (traffic or power interruption, etc.) can cause carefully constructed facilities to be partially or completely destroyed, which may result in higher recourse/mitigation costs. Even minor disruptions can have a significant impact on sales growth and stock returns, and it normally takes a long time for companies to recover (Snyder et al. 2016). Therefore, many authors, including Snyder and Daskin (2005), Cui et al. (2010), An et al. (2014), and Zhang et al. (2015), suggest considering disruptions and the corresponding recourse operations in the system design phase.

Several probability-based models have been proposed (Snyder and Daskin 2005, 2006, Cui et al. 2010, Chen et al. 2011, Shen et al. 2011, Teimuory et al. 2013, Qin et al. 2013, Xie et al. 2015). However, in many situations, it is impossible to obtain or predict precise probability information; there may be insufficient historical data or no accurate forecasting method (An et al. 2014, Snyder et al. 2016). For instance, it is difficult to predict earthquakes. Robust optimization (RO) has been proposed to deal with data uncertainty; it does not require probability information because uncertainty sets are employed to capture randomness. It derives solutions that are robust to any disruptions within the set. The static RO method makes decisions here and now, which could be overly conservative and costly. However, the two-stage RO approach is able to generate less conservative solutions, because it makes recourse decisions based on observed information. Therefore, it has been used to model unit commitment problems in the power industry (An and Zeng 2015), location–transportation problems (Zeng and Zhao 2013), and p-median facility location problems (An et al. 2011).

In this paper, we use a two-stage RO scheme for a network design problem that considers disruption. In the first stage, we make location decisions based on any realization in the uncertainty set; and in the second stage we make recourse decisions based on the first-stage location decisions and the revealed uncertainty. Our study makes the following contributions:

(i) To the best of our knowledge, this paper is the first to solve the reliable LNDP using a two-stage RO approach, which is able to produce less conservative solutions.

(ii) The RO model can be extended to include multiple uncertainty sets and impose upper bounds on the worst-case performance of these sets. It can also be extended to partial disruptions.

(iii) We present an exact algorithm that outperforms the Benders decomposition (BD) method. We present management insights based on the numerical results.

The rest of this paper is organized as follows. Section 2 reviews the literature. Section 3 describes our problem and presents three two-stage RO models. Section 4 introduces an exact algorithm for the models, and Section 5 presents the numerical results. Section 6 concludes the paper and suggests future research directions.

2 Literature review

Supply chain disruption is not a new concept; it has existed as long as the supply chain itself. However, in recent years it has received increasing attention. Snyder et al. (2016) give four reasons for the explosion of interest: (1) high-profile events, such as the 9/11 terrorist attack in the United States and the Japanese earthquake, have brought disruption to the forefront of public attention; (2) the "just in time" concept leaves little room for adjustment, and this significantly exacerbates the impact of disruption; (3) with the development of the global supply chain, suppliers are more integrated and some are located in economically or politically unstable regions; (4) as with any other maturing research area, scholars study this topic because of its high profile.

Drezner (1987) was the first to present mathematical models for location problems with unreliable facilities. In the unreliable p-median problem (PMP) a facility has a given probability of becoming inactive; in the (p, q)-center problem p facilities need to be built and at most q of them will fail simultaneously. Snyder and Daskin (2005) study the reliable PMP and the reliable uncapacitated fixed-charge location problem (UFLP), assuming that each facility that can fail has the same failure probability. Cui et al. (2010) investigate the reliable UFLP and assume that each facility has a site-dependent failure probability. Li and Ouyang (2010) further suppose that the facilities are subject to spatially correlated disruptions. Lim et al. (2010) consider a facility location problem with two types of facilities: unreliable facilities may fail with a probability in the failure state; and reliable facilities will not fail but have higher fixed costs. Rayat et al. (2017) solve a multi-product and multi-period reliable location-inventory-routing problem, where an unreliable distribution center (DC) has a possibility to be disrupted in each period. Farahani et al. (2017) consider a multi-product location-inventory problem. They assume that product k may be out of stock when facility j is partially disrupted, and customers can purchase substitute products from facility j or try another facility to obtain the same product. For more details on the reliable facility location problem see Snyder et al. (2016) and Sawik et al. (2018).

Although supply chain disruption has received extensive attention, research into disruption in the context of network design is scarce (Snyder et al. 2016). The reliable LNDP extends the reliable facility location problem by considering multiple echelons and allowing transshipment nodes in addition to supplier and demand nodes. Both the supplier and transshipment nodes can be destroyed. It also considers facility capacities.

Snyder et al. (2006) propose several scenario-based models (each scenario has an occurrence probability) for designing supply chains that are resilient to disruption. They first present a reliable network design model for a network that will be built from scratch. For existing networks, they provide fortification models and indicate that the reliability of the existing facilities can be enhanced by investing in protection and security measures. Peng et al. (2011) study a reliable LNDP with a $p-$robustness criterion, the objective of which is to minimize the nominal cost. They propose a scenario-based mixed-integer programming (MIP) model and develop a hybrid genetic algorithm. In their numerical tests they randomly generate several scenarios, where each facility has a 10% probability of becoming disabled. Azad et al. (2013) consider a capacitated supply chain network design (SCND) model in which both the facilities and the transportation network have a given probability of disruption. They assume that the facilities are partially destroyed when disruptions occur and that the customers of a disrupted DC are not assigned to other DCs; instead, the capacity lost at the disrupted DC is replenished from non-disrupted DCs. They formulate a linear MIP model and propose a modified BD method. Shishebori et al. (2014) study a reliable facility location/network design problem with a constraint on the maximum allowable failure cost. The facilities and network links are assumed to be uncapacitated. At most one facility fails at a time, and the demand nodes served by the disrupted facility must be reallocated to the nearest surviving facility. The objective is to minimize the transportation cost. Upper bounds are imposed on the investment in facility location and link construction. The authors use a heuristic to solve a MIP model. Rezapour et al. (2017) study the influence of disruptions on the competitiveness of supply chains and propose three policies to mitigate the disruption risk.

RO captures data randomness via uncertainty sets. An et al. (2014) use two-stage RO to model both the uncapacitated and capacitated PMP with disruption considerations, and they minimize the weighted sum of the normal cost and the worst-case cost. Zeng and Zhao (2013) construct a two-stage RO model for the location–transportation problem with demand uncertainty. Baron et al. (2011) apply RO to a multi-period fixed-charge network location problem with uncertain demands, and they explore the influence of various uncertainty sets on the system decisions. Pishvaee et al. (2011) use RO to handle the uncertainty of the input data in a closed-loop SCND problem. Parvaresh et al. (2014) apply RO to a reliable hub network design problem.

Several algorithms have been developed for two-stage RO models: the approximation algorithm (Atamtürk and Zhang 2007, Bertsimas et al. 2011), BD (Jiang et al. 2012, Gabrel et al. 2014), and column-and-constraint-generation (C&CG; Zeng and Zhao 2013, An et al. 2014, An and Zeng 2015).

For approximation algorithms, the second-stage decisions should be simple functions or have special characteristics to ensure that the problem is manageable. The computational time of the BD method increases significantly with increasing instance size. However, the C&CG algorithm has proved to be efficient for two-stage RO models (Zeng and Zhao 2013). Therefore, we use this algorithm, and we develop an enhancement technique to further improve its computational efficiency.

Thus, our paper differs from those on the reliable PMP and UFLP by considering multiple echelons and facility capacities. Specifically, our work differs from An et al. (2015) and An and Zeng (2015) in following aspects: (1) An et al. (2015) focus on analyzing the structural properties of the robust PMP and exploring the modeling capability of two-stage RO by considering partial disruption and demand changes (this is possible because the facility set and the customer set is the same in their paper). However, we extend the basic RO scheme to include multiple uncertainty sets to characterize decision makers' conservative level. And the model in An et al. (2015) can be recognized as a special case of this modeling scheme. We also introduce upper bounds for the worst-case performance, and this modeling framework can be used as a decision support tool for system expansion with reliability considerations. Our numerical tests demonstrate that the new models can generate less conservative solutions. (2) An and Zeng (2015) present robust unit commitment models, where the load is subject to interval uncertainty. They focus on building the connection between the robust models and the stochastic models. However, we apply the modeling scheme to a reliable LNDP and explain the connections and differences among various models. Extensive numerical tests are conducted to study the conservativeness of different models and the price of robustness. Values of key parameters are also analyzed to provide insights for supply chain decision-makers.

3 Model formulation

In this section, we first introduce our notation. We then present a basic two-stage RO model for the reliable LNDP and explore the modeling capability of two-stage RO by describing two variants of the basic model: the *expanded robust* LNDP model and the *risk-constrained robust* LNDP model.

3.1 Notation

3.1.1 Parameters

Consider a general network $(\mathcal{V}, \mathcal{A})$. Let $\mathcal{V}_{\mathcal{S}}$, $\mathcal{V}_{\mathcal{T}}$, and $\mathcal{V}_{\mathcal{D}}$ be the sets of supply, transshipment, and demand nodes. Define $\mathcal{V}_0 = \mathcal{V}_{\mathcal{S}} \cup \mathcal{V}_{\mathcal{T}}$ to be the set of facilities for which open/close decisions are required, and $\mathcal{V} = \mathcal{V}_0 \cup \mathcal{V}_\mathcal{D}$.

The other parameters are as follows:

- f_j = fixed cost to open facility $j \in \mathcal{V}_0$
- c_{ij} = unit transportation cost on arc $(i,j) \in \mathcal{A}$
- Q_j = capacity of facility $j \in \mathcal{V}_0$
- b_j = supply of node $j \in \mathcal{V}$; $b_j \geq 0$ if $j \in \mathcal{V}_\mathcal{S}$, $b_j = 0$ if $j \in \mathcal{V}_\mathcal{T}$, and $b_j \leq 0$ if $j \in \mathcal{V}_\mathcal{D}$.
- θ_i = unit penalty cost for unsatisfied demand at node $i \in \mathcal{V}_\mathcal{D}$

3.1.2 Decision variables

The decisions are made in two stages. We make location decisions at the first stage. After the disruption, we implement recourse or mitigation operations at the second stage: reassigning demand nodes to surviving facilities to ensure system reliability. We use $\mathbf{y}$ and $\mathbf{x}$, $\mathbf{u}$ to represent the first- and second-stage decision variables respectively, i.e.,

- $y_j = 1$ if facility $j \in \mathcal{V}_0$ is opened in the first stage, $y_j = 0$ otherwise
- x_{ij} = product flow on arc (i,j) in a specific disruptive scenario
- u_i = unsatisfied demand at node $i \in \mathcal{V}_\mathcal{D}$ in a specific disruptive scenario

Our two-stage robust optimization model uses a budgeted uncertainty set to describe possible disruptive scenarios without requiring any probabilistic information. We assume that at most m facilities fail simultaneously:

$$A = \left\{ \mathbf{z} \in \{0,1\}^{|\mathcal{V}_0|} : \sum_{j \in \mathcal{V}_0} z_j \leq m \right\}, \tag{1}$$

where $z_j = 1$ if facility j is disrupted and $z_j = 0$ otherwise.

3.2 Formulations

We first present the basic two-stage RO model, in which only one uncertainty set is considered. Then we introduce two models with multiple uncertainty sets.

3.2.1 Basic two-stage RO model

We formulate the basic two-stage RO LNDP model as follows, where $\mathcal{V}_i^+ = \{j \in \mathcal{V} | (i,j) \in \mathcal{A}\}$ and $\mathcal{V}_i^- = \{j \in \mathcal{V} | (j,i) \in \mathcal{A}\}$.

RO-LNDP$_0$:

$$\min_{\mathbf{y}} \sum_{j\in\mathcal{V}_0} f_j y_j + \max_{\mathbf{z}\in A} \min_{\mathbf{x},\mathbf{u}\in S(\mathbf{y},\mathbf{z})} \Big(\sum_{(i,j)\in\mathcal{A}} c_{ij}x_{ij} + \sum_{j\in\mathcal{V}_\mathcal{D}} \theta_j u_j\Big) \tag{2}$$

s.t.

$$y_j \in \{0,1\} \quad \forall j \in \mathcal{V}_0 \tag{3}$$

Here $S(\mathbf{y},\mathbf{z}) = \{$

$$\sum_{i\in\mathcal{V}_j^+} x_{ji} \le b_j \quad \forall j \in \mathcal{V}_\mathcal{S} \tag{4}$$

$$\sum_{i\in\mathcal{V}_j^+} x_{ji} = \sum_{i\in\mathcal{V}_j^-} x_{ij} \quad \forall j \in \mathcal{V}_\mathcal{T} \tag{5}$$

$$\sum_{i\in\mathcal{V}_j^-} x_{ij} + u_j = -b_j \quad \forall j \in \mathcal{V}_\mathcal{D} \tag{6}$$

$$\sum_{i\in\mathcal{V}_j^+} x_{ji} \le (1-z_j)Q_j y_j \quad \forall j \in \mathcal{V}_0 \tag{7}$$

$$x_{ij} \ge 0 \quad \forall (i,j) \in \mathcal{A} \tag{8}$$

$$u_j \ge 0 \quad \forall j \in \mathcal{V}_\mathcal{D}\} \tag{9}$$

The objective function in (2) minimizes the cost of the worst-case scenario. The *max* operator represents the disruptive scenario in A that generates the largest recourse cost, given the facility locations $\mathbf{y}$. The *min* operator identifies the least costly solution, and the set $S(\mathbf{y},\mathbf{z})$ represents possible recourse operations. Constraints (4)–(6) are the product flow conservation equations for all nodes. Constraints (7) ensure that when a facility is open and functional the flow does not exceed its capacity, and they prohibit any flow when it is closed or destroyed. Constraints (3), (8), and (9) define the variable types.

3.2.2 Expanded two-stage RO model

Two key factors influence the solution of the two-stage RO: the uncertainty set and the worst-case performance. A small uncertainty set cannot adequately capture the random factor; a large set leads to solutions that are costly and overly conservative. To deal with this, An and Zeng (2015) suggest using multiple uncertainty sets and assigning different weights to the worst-case performances of these sets. Our RO-LNDP$_0$ model can be extended to multiple uncertainty sets as follows:

RO-LNDP$_1$:

$$\min_{\mathbf{y}} \sum_{j\in\mathcal{V}_0} f_j y_j + \sum_{k=1}^{K} \rho_k \left(\max_{\mathbf{z}\in A_k} \min_{\mathbf{x},\mathbf{u}\in S_k(\mathbf{y},\mathbf{z})} \big(\sum_{(i,j)\in\mathcal{A}} c_{ij}x_{ij} + \sum_{j\in\mathcal{V}_\mathcal{D}} \theta_j u_j \big) \right) \tag{10}$$

where $\mathbf{y}$ and $S_k(\mathbf{y},\mathbf{z})$ are defined by constraints (3) and (4)–(9), respectively. In the objective function (10), A_k denotes the kth uncertainty set, with weight ρ_k ($0 \le \rho_k \le 1$ and $\sum_k \rho_k = 1$).

3.2.3 Risk-constrained two-stage RO model

Another way to guarantee the quality of the solutions is to impose upper bounds on the worst-case performance of the uncertainty sets. Then any solution that is feasible with respect to these uncertainty sets provides a performance guarantee. We extend our RO-LNDP$_0$ model to introduce the *risk-constrained robust* LNDP model, where ξ_k is the performance restriction on uncertainty set $A_k (k = 1, 2, ..., |K|)$, and x_{ij0} and u_{j0} are the normal disruption-free decisions.

RO-LNDP$_2$:

$$\min \sum_{j\in\mathcal{V}_0} f_j y_j + \sum_{(i,j)\in\mathcal{A}} c_{ij}x_{ij0} + \sum_{j\in\mathcal{V}_\mathcal{D}} \theta_j u_{j0} \tag{11}$$

s.t.

$$\sum_{i\in\mathcal{V}_j^+} x_{ji0} \le b_j \quad \forall j \in \mathcal{V}_\mathcal{S} \tag{12}$$

$$\sum_{i\in\mathcal{V}_j^+} x_{ji0} = \sum_{i\in\mathcal{V}_j^-} x_{ij0} \quad \forall j \in \mathcal{V}_\mathcal{T} \tag{13}$$

$$\sum_{i\in\mathcal{V}_j^-} x_{ij0} + u_{j0} = -b_j \quad \forall j \in \mathcal{V}_\mathcal{D} \tag{14}$$

$$\sum_{i\in\mathcal{V}_j^+} x_{ji0} \le Q_j y_j \quad \forall j \in \mathcal{V}_0 \tag{15}$$

$$\max_{\mathbf{z}\in A_k} \min_{\mathbf{x},\mathbf{u}\in S_k(\mathbf{y},\mathbf{z})} \sum_{(i,j)\in\mathcal{A}} c_{ij}x_{ij} + \sum_{j\in\mathcal{V}_\mathcal{D}} \theta_j u_j \le \xi_k, \quad k = 1,\dots,|K| \tag{16}$$

$$y_j \in \{0,1\} \quad \forall j \in \mathcal{V}_0 \tag{17}$$

$$x_{ij0} \ge 0 \quad \forall (i,j) \in \mathcal{A} \tag{18}$$

$$u_{j0} \ge 0 \quad \forall j \in \mathcal{V}_\mathcal{D} \tag{19}$$

where $S_k(\mathbf{y},\mathbf{z})$ is defined by (4)–(9).

For comparison purposes, in Appendix A we formulate the generic LNDP (G-LNDP) that ignores disruptions. In Appendix B, we give the formulation of the stochastic programming (SP) model.

3.2.4 Summary of models and another extension

The connections and differences among models are shown in Figure 1 and Table 1. The G-LNDP model is a special case of the RO-LNDP$_0$ model with $m = 0$, and also a special case of the RO-LNDP$_1$ model with $K = 1$ and $m = 0$. The RO-LNDP$_1$ model reduces to the RO-LNDP$_0$ model when $K = 1$ and $m > 0$. Furthermore, An and Zeng (2015) have proved that the RO-LNDP$_1$ model is equivalent to the SP model when the uncertainty sets are individual scenarios. The RO-LNDP$_2$ model reduces to the G-LNDP model when the performance bound ξ_k is sufficiently large.

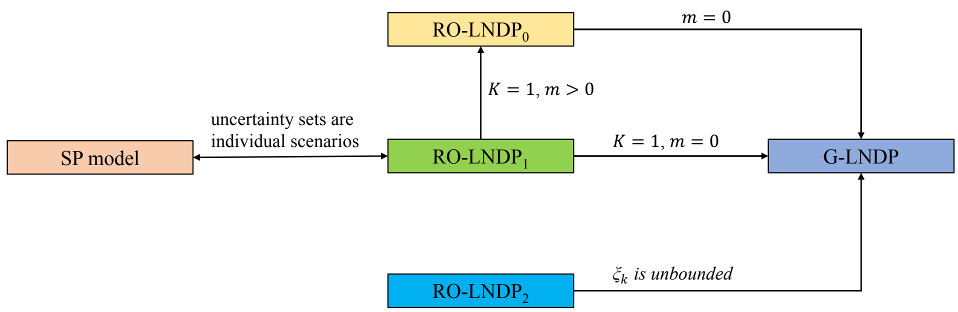


Figure 1: Connections among models

From Table 1, our three two-stage robust models differ in the number of uncertainty sets and the objective function. The basic RO model has one uncertainty set, and the other models have multiple uncertainty sets. The objective of the basic RO model is to minimize the cost of the worst-case scenario in the uncertainty set. The expanded RO model identifies the worst-case scenario in each uncertainty set and minimizes the weighted sum. The risk-constrained RO model minimizes the cost of the normal disruption-free situation.

Table 1: Model comparison

Model	Possibility information required?	Uncertainty set/scenarios	Objective (minimize cost)
Generic LNDP	Not applicable	Not applicable	Normal disruption-free case
Basic RO	No	One	Worst-case
Expanded RO	No	Multiple	Weighted sum of multiple worst-cases
Risk-constrained RO	No	Multiple	Normal disruption-free case
Stochastic programming	Yes	Multiple	Weighted sum of multiple scenarios

Another extension of our work allows partial disruption in which a damaged facility can still satisfy part of the demand. We introduce a parameter δ_j ($0 < \delta_j \leq 1$) to represent the change in facility j's capacity in a disruptive scenario, and constraints (7) become

$$\sum_{i \in \mathcal{V}_j^+} x_{ji} \leq (1 - \delta_j z_j) Q_j y_j \quad \forall j \in \mathcal{V}_0 \tag{20}$$

4 Solution method

Two-stage RO models are usually difficult to solve (Ben-Tal et al. 2004). Although BD can be used to find optimal solutions for the second-stage problem (if it is linear), it is not efficient for large problems. Recently, the C&CG algorithm has been developed to solve two-stage RO models. It has performed well on unit commitment problems (Zhao and Zeng 2012, An and Zeng 2015) and p-median facility location problems (An et al. 2014). We use this algorithm, and we introduce an enhancement strategy to further improve its computational efficiency.

4.1 Implementation of the C&CG algorithm

The C&CG algorithm is implemented in a master-subproblem framework. In the subproblem, the solution for the master problem (i.e., the location decision) is known and we solve the remaining max–min problem. Since unmet demand will be penalized in disruptive scenarios, the second-stage problem is always feasible. Therefore, we find its dual and obtain a max–max problem, which can be merged into a maximization problem. We describe our algorithm for RO-LNDP$_0$; the other RO models can be solved with minor modifications.

4.1.1 Dual of the second-stage problem

We introduce the dual variables $\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}$, and $\boldsymbol{\pi}$ for constraints (4), (5), (6), and (7), respectively. The resulting dual problem is as follows:

<u>NonL-SubP:</u>

$$\max \sum_{j\in\mathcal{V}_\mathcal{S}} b_j\alpha_j - \sum_{j\in\mathcal{V}_\mathcal{D}} b_j\gamma_j + \sum_{j\in\mathcal{V}_0}(1-z_j)Q_j\hat{y}_j\pi_j \tag{21}$$

s.t.

$$\alpha_i + \beta_j + \pi_i \le c_{ij} \quad \forall i\in\mathcal{V}_\mathcal{S}, j\in\mathcal{V}_\mathcal{T}\cap\mathcal{V}_i^+ \tag{22}$$

$$\alpha_i + \gamma_j + \pi_i \le c_{ij} \quad \forall i\in\mathcal{V}_\mathcal{S}, j\in\mathcal{V}_\mathcal{D}\cap\mathcal{V}_i^+ \tag{23}$$

$$-\beta_i + \gamma_j + \pi_i \le c_{ij} \quad \forall i\in\mathcal{V}_\mathcal{T}, j\in\mathcal{V}_\mathcal{D}\cap\mathcal{V}_i^+ \tag{24}$$

$$\gamma_j \le \theta_j \quad \forall j\in\mathcal{V}_\mathcal{D} \tag{25}$$

$$\alpha_j \le 0 \quad \forall j\in\mathcal{V}_\mathcal{S} \tag{26}$$

$$\pi_j \le 0 \quad \forall j\in\mathcal{V}_0 \tag{27}$$

Since $\pi_j \le 0$ and $z_j \in \{0,1\}$, the nonlinear term $z_j\pi_j$ is the product of a binary variable and a continuous variable. We can linearize it by introducing a new continuous variable $w_j = z_j\pi_j$ and using a big-M method, with the following constraints:

$$w_j \ge \pi_j \tag{28}$$

$$w_j \ge -Mz_j \tag{29}$$

$$w_j \le \pi_j + M(1-z_j) \tag{30}$$

$$w_j \le 0 \tag{31}$$

The value of M can be set as follows: For each facility $j\in\mathcal{V}_0$ and demand node $i\in\mathcal{V}_\mathcal{D}$, define c'_{ji} as the minimal cost from j to i, i.e., $c'_{ji} = \min\{c_{ji}, c_{jk}+c_{ki}\}$, where k is a transshipment node. If there is no arc between facility j and demand node i, then $c_{ji} = +\infty$. Define $M'_j = \max_{i\in\mathcal{V}_\mathcal{D}}\{\theta_i - c'_{ji}\}$ and $M''_j = \max_{i\in\mathcal{V}_j^+}\{c_{ji}\}$. Then we set

$$-\pi_j \le \max\{M'_j, M''_j\} = M_j \quad j\in\mathcal{V}_0 \tag{32}$$

Therefore, the linearized subproblem is:

<u>L-SubP:</u>

$$\chi = \max \sum_{j\in\mathcal{V}_\mathcal{S}} b_j\alpha_j - \sum_{j\in\mathcal{V}_\mathcal{D}} b_j\gamma_j + \sum_{j\in\mathcal{V}_0} Q_j\hat{y}_j(\pi_j - w_j) \tag{33}$$

subject to constraints (22)–(31).

4.1.2 Framework of the C&CG algorithm

We now describe the framework of the C&CG algorithm and present the formulation of the master problem (MP), which will be solved iteratively. At each iteration r, we identify a worst-case scenario $\hat{\mathbf{z}}^r$ by solving the linearized subproblem. Then we create the recourse variables $(\mathbf{x}^r, \mathbf{u}^r)$ and the corresponding constraints, as well as this specific scenario, and add them to the MP. Let LB and UB be the lower and upper bounds, $Gap = (UB - LB)/UB$, and let the optimality tolerance be ϵ. The C&CG algorithm is as follows:

(1) Set LB = -∞, $UB = +\infty$, and $r = 0$.

(2) Take any arbitrary $\hat{\mathbf{y}} \in \{0,1\}^{|\mathcal{V}_0|}$ as initial solution.

(3) Solve the linearized subproblem with regards to $\hat{\mathbf{y}}$ to identify the worst-case scenario $\hat{\mathbf{z}}$. Update $r = r + 1$. Create recourse variables $(\mathbf{x}^r, \mathbf{u}^r)$ and corresponding constraints, and add them to the following MP.

MP:

$$\min \sum_{j \in \mathcal{V}_0} f_j y_j + \phi \tag{34}$$

s.t.

$$\phi \geq \sum_{(i,j) \in \mathcal{A}} c_{ij} x_{ij}^l + \sum_{j \in \mathcal{V}_\mathcal{D}} \theta_j u_j^l \quad \forall l = 1, 2, ..., r \tag{35}$$

$$\sum_{i \in \mathcal{V}_j^+} x_{ji}^l \leq b_j \quad \forall j \in \mathcal{V}_\mathcal{S} \quad \forall l = 1, 2, ..., r \tag{36}$$

$$\sum_{i \in \mathcal{V}_j^+} x_{ji}^l = \sum_{i \in V_j^-} x_{ij}^l \quad \forall j \in \mathcal{V}_\mathcal{T} \quad \forall l = 1, 2, ..., r \tag{37}$$

$$\sum_{i \in \mathcal{V}_j^-} x_{ij}^l + u_j^l = -b_j \quad \forall j \in \mathcal{V}_\mathcal{D} \quad \forall l = 1, 2, ..., r \tag{38}$$

$$\sum_{i \in \mathcal{V}_j^+} x_{ji}^l \leq (1 - z_j^l) Q_j y_j \quad \forall j \in \mathcal{V}_0 \quad \forall l = 1, 2, ..., r \tag{39}$$

$$y_j \in \{0,1\} \quad \forall j \in \mathcal{V}_0 \tag{40}$$

$$x_{ij}^l \geq 0 \quad \forall (i,j) \in \mathcal{A} \quad \forall l = 1, 2, ..., r \tag{41}$$

$$u_j^l \geq 0 \quad \forall j \in \mathcal{V}_\mathcal{D} \quad \forall l = 1, 2, ..., r \tag{42}$$

(4) Iterate until the algorithm terminates:

(i) Solve the MP to find an optimal solution $(\hat{\mathbf{y}}, \phi)$; set LB to the optimal value of the MP.

(ii) Solve the subproblem with regards to $\hat{\mathbf{y}}$ to identify the worst-case scenario $\hat{\mathbf{z}}$. Update $UB = \min\left\{UB, \sum_{j\in\mathcal{V}_0} f_j y_j^r + \chi^r\right\}$.

(iii) If $Gap \leq \epsilon$, terminate; otherwise, update $r = r + 1$ and create the recourse variables and corresponding constraints. Add them to the MP and go to step (i).

We also implement BD, in which a single cutting plane

$$\phi \geq \sum_{j\in\mathcal{V}_\mathcal{S}} b_j\alpha_j^r - \sum_{j\in\mathcal{V}_\mathcal{D}} b_j\gamma_j^r + \sum_{j\in\mathcal{V}_0} (1 - z_j^r)Q_j y_j \pi_j^r \tag{43}$$

is iteratively added to the MP, which carries only the first-stage decision variable $\mathbf{y}$.

Remarks: (1) for the RO-LNDP$_1$ model, the objective function of the MP becomes

$$\min \sum_{j\in\mathcal{V}_0} f_j y_j + \sum_{k=1}^{K} \rho_k\phi_k \tag{44}$$

where ϕ_k corresponds to the kth uncertainty set. For each uncertainty set, we need to solve a subproblem to identify the worst-case scenario, and we add its recourse variables and corresponding constraints to the MP.

(2) For the RO-LNDP$_2$ model, at each iteration we solve a subproblem for each uncertainty set, to check whether its worst-case performance violates the bound. Once there exists such scenarios, we add all the identified worst-case scenarios to the MP until all the performance bounds are respected.

4.2 Algorithm enhancement

In this section we introduce a *variable fixing technique* to improve the algorithm's performance. This technique has been shown to be efficient in reducing the computational burden for facility location problems (Snyder and Daskin 2005, Contreras et al. 2011, Zhang et al. 2016). We generalize it to solve the reliable LNDP. The idea is as follows:

Let $\hat{\mathbf{y}}$ be the incumbent solution with corresponding upper bound UB'.

- If $\hat{y}_j = 0$: we add an additional constraint $y_j = 1$ to the MP and solve it to optimality; if its optimal value is larger than UB', then we fix y_j to 0 in the MP.
- If $\hat{y}_j = 1$: we add an additional constraint $y_j = 0$ to the MP and solve it to optimality; if its optimal value is larger than UB', then we fix y_j to 1 in the MP.

To understand this technique, imagine that we choose to branch on variable y_j with two branches $y_j = 1$ and $y_j = 0$ in the branch-and-bound tree. Clearly, the solutions obtained from both nodes are lower bounds for their children nodes, respectively. If the generated solution is larger than UB', we prune the tree at the corresponding node, because all of its children nodes will generate solutions that are larger than UB'.

Fixing some y_j reduces the feasible space of the MP, which can help improve computational efficiency. On the other hand, each time a new constraint with $y_j = 1$ or 0 is added, and this might increase the computational time, especially when the number of facilities is large. Therefore, (1) after adding the new constraint, we solve the corresponding linear relaxation and get a lower bound, and we compare this bound with UB' ("C&CG LP"); (2) we solve the MP with the new constraint to optimality and compare the optimal value with UB' ("C&CG Optimal").

5 Numerical experiments and analyses

In this section, we present the instances, discuss our numerical tests, analyze the influence of the parameters, and present some insights. The algorithm is coded in the C# programming language and run on a PC with a 2.53 GHz Intel Core Dual Processor and 3 GB of memory. The MP and subproblem are solved using Gurobi 6.0.

5.1 Instances

We randomly generate instances of different sizes. The method is based on that of Peng et al. (2011), with a few modifications. The instances are labeled "$d-|\mathcal{V}_{\mathcal{S}}|-|\mathcal{V}_{\mathcal{T}}|-|\mathcal{V}_{\mathcal{D}}|$," where d is the edge density (20%, 30%, or 50%) and $|\mathcal{V}_{\mathcal{S}}|$, $|\mathcal{V}_{\mathcal{T}}|$, and $|\mathcal{V}_{\mathcal{D}}|$ are the number of supply, transshipment, and demand nodes. The number of nodes ranges from 60 to 100.

For each demand node $j \in \mathcal{V}_{\mathcal{D}}$, the unmet-demand penalty is 1500, and the demand b_j is drawn uniformly from $[-110, -50]$. Let $S_b = -\sum_{j\in\mathcal{V}_{\mathcal{D}}} b_j$ be the sum of all the demands (the negative sign appears because $b_j \leq 0$); define $\bar{s} = \frac{S_b}{|\mathcal{V}_{\mathcal{S}}|}$ and $\bar{c} = \frac{S_b}{|\mathcal{V}_{\mathcal{T}}|}$.

For each facility node $j \in \mathcal{V}_0$, the fixed cost is drawn uniformly from $[5000, 15000]$; if $j \in \mathcal{V}_{\mathcal{S}}$, then its capacity Q_j is drawn uniformly from $[1.5\bar{s}, 2.5\bar{s}]$ and its supply b_j is the same as its capacity; if $j \in \mathcal{V}_{\mathcal{T}}$, then its capacity Q_j is drawn uniformly from $[1.5\bar{c}, 2.5\bar{c}]$ and its supply b_j is 0.

The arcs are constructed based on the probability specified by the edge density. In detail, for two nodes i, j ($i \in \mathcal{V}_{\mathcal{S}}, j \in \mathcal{V}_{\mathcal{T}}$ or $i \in \mathcal{V}_{\mathcal{S}}, j \in \mathcal{V}_{\mathcal{D}}$ or $i \in \mathcal{V}_{\mathcal{T}}, j \in \mathcal{V}_{\mathcal{D}}$) and edge density d, we generate a random number $r \in [0, 1]$. If $r \leq d$, then we construct an arc between i and j. The unit transportation cost of each arc $(i, j) \in \mathcal{A}$ is drawn uniformly from $[1, 500]$.

5.2 Algorithm performance

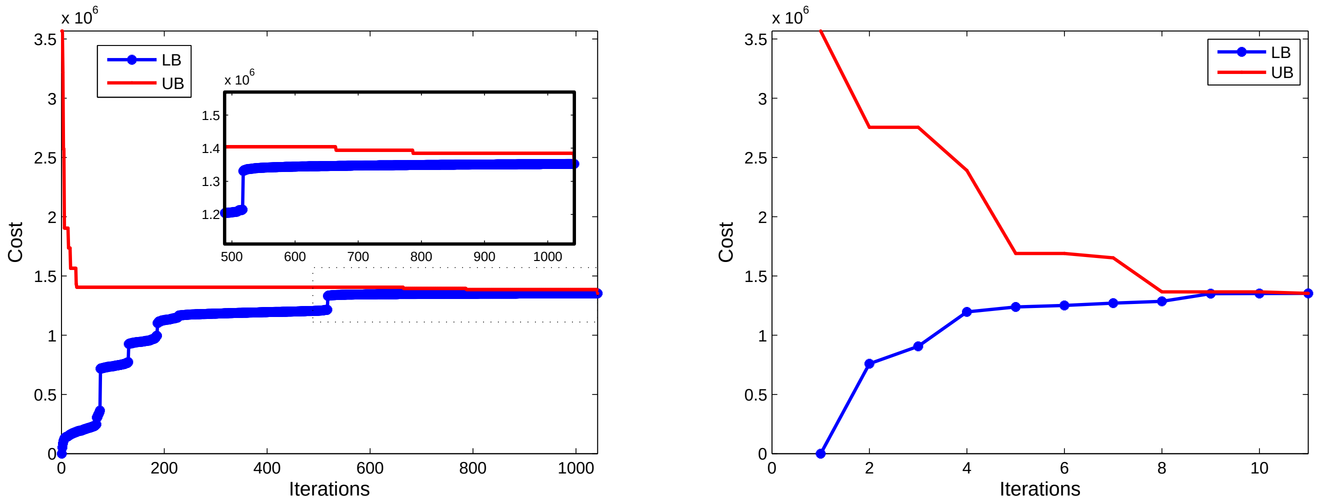
We now evaluate the performance of the algorithms. We set the maximal number of facilities that can fail simultaneously to 5, i.e., $m = 5$ (we arbitrarily choose a large value for m to test the efficiency of the C&CG algorithm). The facilities are destroyed completely when disruptions occur (i.e., $\delta_j = 1, \forall j \in \mathcal{V}_0$). The model is RO-LNDP$_0$, and the results are shown in Table 2. The optimality tolerance ϵ is set to 10^{-6} and the time limit is 10800 seconds (when this limit is reached, the current iteration will be completed). In Table 2, the results in the "C&CG No Fix" columns are obtained by using the C&CG algorithm without enhancement. The column Iter gives the number of iterations; Time indicates the computational time in seconds; and Gap is the relative percentage gap between the upper and lower bounds.

Table 2 shows that for most instances (9 out of 13), the C&CG algorithm is hundreds of times faster than BD, and the number of iterations is much lower. Within the time limit, BD can find optimal solutions for just 3 instances. In contrast, the C&CG algorithm is able to generate optimal solutions for most of the instances, and C&CG LP finds the optimal solution for all the instances. Therefore, the C&CG algorithm outperforms BD in both computational time and solution quality.

To further demonstrate the superiority of C&CG LP, Figure 2 shows the convergence curves of the two algorithms for the instance 20%-10-20-30. In Figure 2(a), the gap between the lower and upper bounds reduces slowly and does not reach zero even after 1000 iterations; the C&CG LP algorithm finds the optimal solution after 11 iterations. The instances 50%-20-20-30, 50%-30-20-30, and 50%-40-20-30 show that the number of supply nodes has a significant impact on the computational efficiency. For these instances, C&CG LP finds optimal solutions within the time limit and performs better than C&CG No Fix and C&CG Optimal. Therefore, for our model comparison and parameter sensitivity analysis, we use C&CG LP.

Table 2: Performance of different algorithms

	Benders				C&CG LP				C&CG No Fix		C&CG Optimal	
Instance	Iter	Time	Gap	UB	Iter	Time	Gap	UB	Time	Gap	Time	Gap
20%-10-20-30	1042	696.84	0.00	1352471	11	5.66	0.00	1352471	5.19	0.00	72.71	0.00
30%-10-20-30	3710	5126.89	0.00	1532355	6	0.94	0.00	1532355	0.71	0.00	11.32	0.00
50%-10-20-30	919	10831.45	78.59	767158	9	5.37	0.00	751133	4.75	0.00	48.14	0.00
50%-**20**-20-30	553	10836.64	74.65	663328	41	4005.28	0.00	627139	9662.08	0.00	10839.47	4.91
50%-**30**-20-30	387	10851.49	65.83	577131	25	6930.84	0.00	532691	12303.52	4.31	12794.40	25.40
50%-**40**-20-30	391	10812.02	57.25	502841	22	8177.58	0.00	421706	11763.53	6.24	18315.67	13.03
30%-10-**30**-30	809	10813.28	35.98	1591183	9	2.86	0.00	1531845	2.76	0.00	46.52	0.00
30%-10-**40**-30	227	10914.11	78.85	1123941	7	4.24	0.00	1095014	3.77	0.00	78.95	0.00
30%-10-**50**-30	105	10910.24	77.63	1317280	8	3.61	0.00	1249269	3.41	0.00	100.41	0.00
20%-10-20-**40**	3504	10804.99	13.06	1960874	19	15.09	0.00	1923711	24.96	0.00	211.40	0.00
20%-10-20-**50**	1546	10808.08	17.94	2265886	14	14.68	0.00	2249769	20.85	0.00	120.54	0.00
20%-10-20-**60**	642	494.06	0.00	2698076	17	16.04	0.00	2698076	19.71	0.00	219.11	0.00
50%-20-30-50	592	10814.97	81.88	951850	32	7440.17	0.00	709762	11930.12	2.75	11707.76	6.08
Average	1110	8824.23	44.74	1331106	17	2047.87	0.00	1282688	3518.87	1.02	4197.42	3.80



(a) Benders decomposition

(b) C&CG

Figure 2: Convergence curves for instance 20%-10-20-30

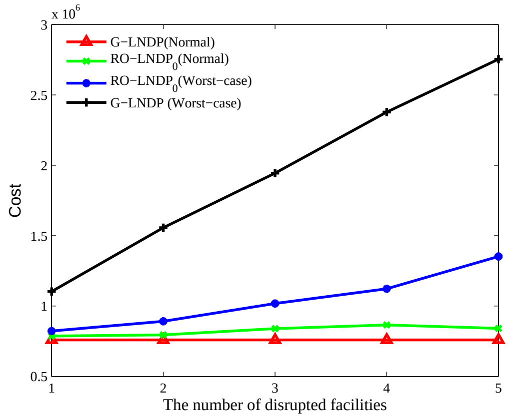
5.3 Impact of reliability

Obviously, if disruptions are not considered (i.e., G-LNDP), the system's normal cost will be lower, but it may be more expensive to implement mitigation/recourse operations when they become necessary. To investigate the impact of reliability on the system cost, we conduct the following experiments:

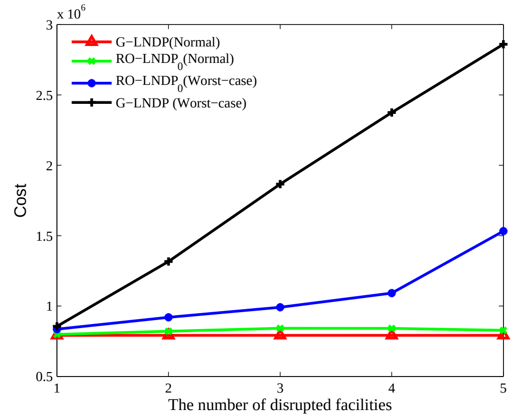
(1) We solve RO-LNDP$_0$ to find the location decision. We then fix this decision and solve a minimum cost flow problem (MCFP) to find the system's normal cost under RO-LNDP$_0$. This indicates the impact of disruptions on the system's normal cost.

(2) We solve G-LNDP and fix the location decision. We then solve the slave problem of the C&CG algorithm to identify the worst-case cost. This indicates the cost of not considering disruptions in advance and handling them as they occur.

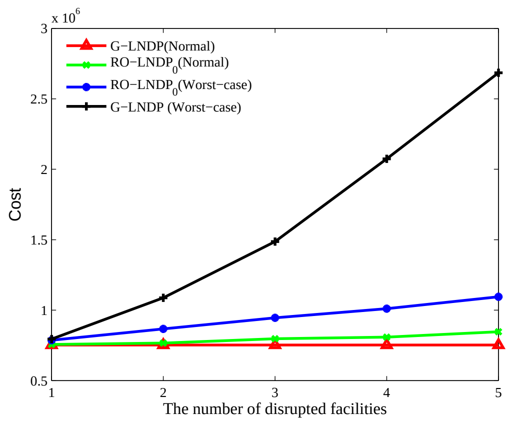
For space considerations, Figure 3 shows the curves of just 6 instances. The results for the other instances are similar. Figure 3 confirms that considering disruptions will increase the normal cost. However, ignoring disruptions during the design phase leads to higher costs when they do occur. As m increases, the deviation of the worst-case cost for RO-LNDP$_0$ and G-LNDP also increases. However, the normal cost of these two models is similar. We conclude that the two-stage RO model gives a considerable decrease of the recourse cost in the worst disruptive situation with only a small increase in the normal cost.



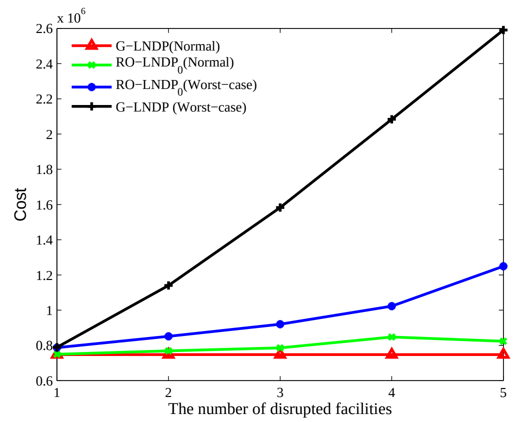
(a) Instance 20%-10-20-30



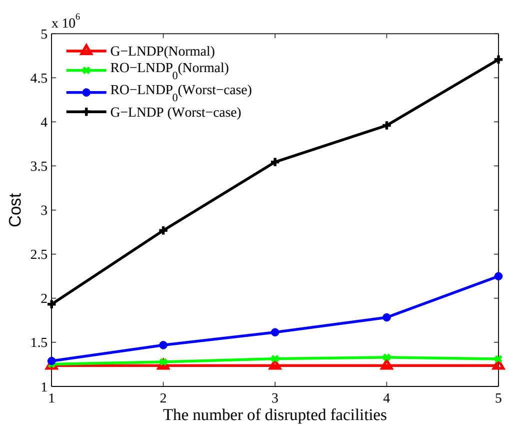
(b) Instance 30%-10-20-30



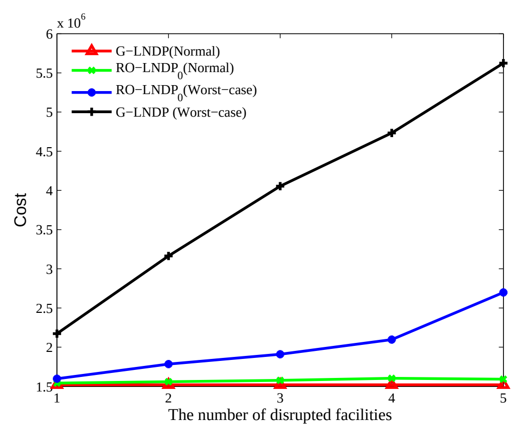
(c) Instance 30%-10-40-30



(d) Instance 30%-10-50-30



(e) Instance 20%-10-20-50



(f) Instance 20%-10-20-60

Figure 3: Impact of reliability

5.4 Model comparison and analyses

In this section, we compare the results of different models.

5.4.1 Basic RO model and expanded RO model

The experimental setup for each model is as follows:

- *Basic two-stage RO model.* We have one uncertainty set with $m = 2$.
- *Expanded two-stage RO model.* We have three uncertainty sets A_0, A_1, and A_2. Set A_0 is the normal disruption-free situation. In sets A_1 and A_2, *exactly* 1 and 2 facilities will fail simultaneously, respectively. For the weight coefficients, we consider two cases: (1) $\rho_0 = 0.8$, $\rho_1 = 0.15$, and $\rho_2 = 0.05$, where the decision-maker believes that in most cases the system will not experience disruption; (2) $\rho_0 = 0.65$, $\rho_1 = 0.25$, and $\rho_2 = 0.10$, where disruption is more likely.

In the following tables, the column Iter gives the number of iterations; FacN. gives the number of opened facilities in the optimal solution; and Time gives the computational time in seconds.

Table 3: Comparison of basic two-stage RO model and expanded two-stage RO model

Instance	Basic two-stage RO model				Expanded two-stage RO model							
					$\rho_0 = 0.8$, $\rho_1 = 0.15$, $\rho_2 = 0.05$				$\rho_0 = 0.65$, $\rho_1 = 0.25$, $\rho_2 = 0.10$			
	Iter	Cost	FacN.	Time	Iter	Cost	FacN.	Time	Iter	Cost	FacN.	Time
20%-10-20-30	15	891019	13	2.55	8	798609	10	21.33	11	813431	13	41.22
30%-10-20-30	5	919641	11	0.53	7	816046	9	10.12	7	827148	9	10.23
50%-10-20-30	6	588292	15	2.71	3	471472	15	1.23	3	483600	15	1.13
30%-10-30-30	6	901721	11	1.83	4	815471	8	4.12	4	827332	9	3.82
30%-10-40-30	6	867244	9	2.28	5	773600	8	8.53	4	783327	9	4.35
30%-10-50-30	6	851158	11	3.64	4	767809	8	9.38	4	778780	9	8.34
20%-10-20-40	8	1323446	14	4.70	6	1072862	10	5.75	6	1104976	13	6.41
20%-10-20-50	5	1467915	14	1.36	6	1280748	13	13.83	5	1295309	13	9.65
20%-10-20-60	8	1783269	14	6.25	13	1572904	13	156.05	9	1592708	13	65.98
Average	7.2	1065967	12.4	2.87	6.2	929947	10.4	25.59	5.9	945179	11.4	16.79

Table 3 shows that the solution under the basic RO model is more conservative than that of the expanded RO model. To hedge against the worst disruptive scenario in the uncertainty set, the basic RO model tends to open more facilities, increasing the cost. In the expanded RO model, where more weight is put on the disruptive scenarios, the cost increases as expected; however, it is still lower than that of the basic RO model.

The computational time is slightly higher for the expanded RO model than the basic RO model. However, with CC&G LP, the expanded model can still be solved to optimality in a short time.

5.4.2 Expanded RO model and stochastic programming model

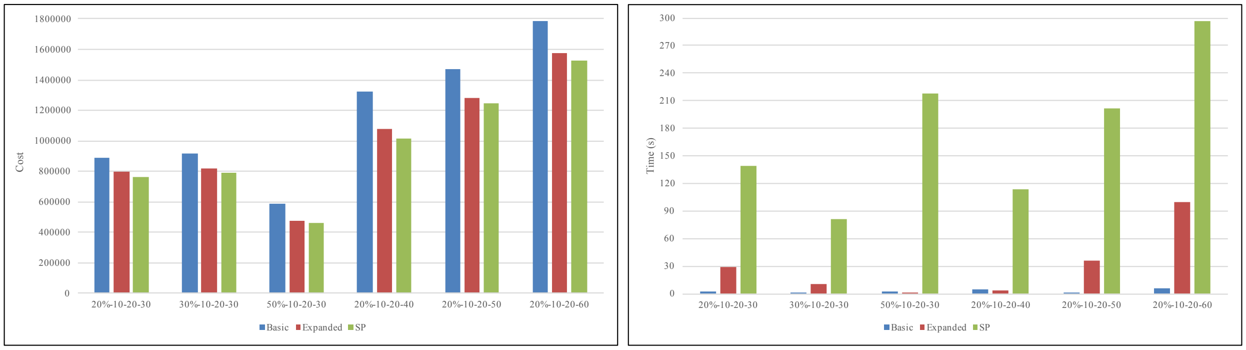
We select instances 20%-10-20-30, 30%-10-20-30, 50%-10-20-30, 20%-10-20-40, 20%-10-20-50, and 20%-10-20-60 for these analyses, and the number of facilities is 30. For the expanded two-stage RO model, we use three uncertainty sets as in Section 5.4.1, and the weight coefficients are $\rho_0 = 0.74$, $\rho_1 = 0.22$, and $\rho_2 = 0.04$. For the SP model, the failure probability of each facility is set to 0.01, which roughly matches the weight of each uncertainty set in the expanded RO model: the probability is 0.740 for the normal disruption-free situation, 0.224 for the scenario with exactly 1 facility disrupted, and 0.033 for the scenario with exactly 2 facilities disrupted. The results are summarized in Table 4.

Table 4: Comparison of expanded RO model and SP model

Instance	Expanded RO model			SP model			Cost gap (%)
	Cost	FacN.	Time	Cost	FacN.	Time	
20%-10-20-30	800822	10	28.74	766073	8	139.41	4.54
30%-10-20-30	817179	9	10.38	792632	7	80.98	3.10
50%-10-20-30	474034	15	1.64	460196	15	218.29	3.01
20%-10-20-40	1076380	10	3.60	1016687	8	113.36	5.87
20%-10-20-50	1281164	13	35.45	1243934	9	201.53	2.99
20%-10-20-60	1572717	12	99.64	1528937	9	296.04	2.86
Average	1003716	11.5	29.91	968076	9.3	174.94	3.73

Table 4 shows that the computational time for the SP model is much higher than that of the expanded RO model. The total cost of the robust model is slightly higher, because it puts all the weight on the worst-case scenarios while the SP model considers all the possible scenarios. On average, the cost increases by 3.73%, which demonstrates that the expanded robust model is not too conservative.

Figure 4 shows that the results from the expanded RO model are between those of the SP model and the basic RO model, in terms of both cost and computational time. This confirms our observations that 1) the expanded RO model reduces to the basic RO model when there is only one uncertainty set; and 2) the expanded RO model becomes the SP model when the uncertainty sets are individual scenarios. We conclude that the expanded two-stage RO model provides a good trade-off among cost,



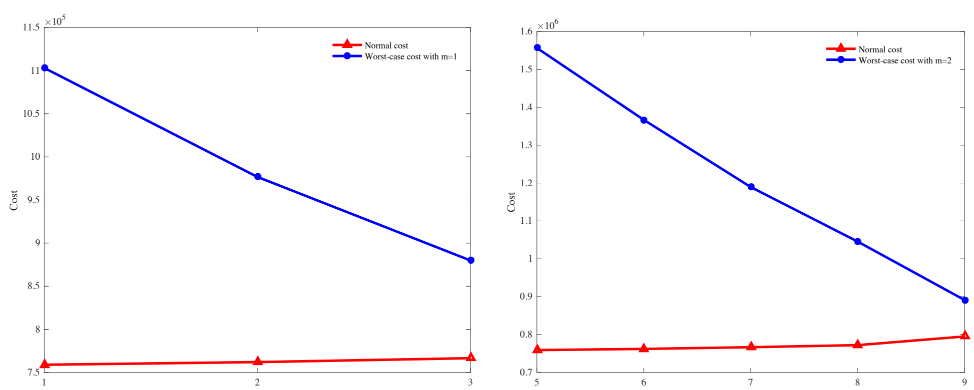
(a) (b)

Figure 4: Comparison among basic RO model, expanded RO model and SP model

risk, and computational burden.

5.4.3 Risk-constrained robust model and generic LNDP model

For the risk-constrained RO model, we use two uncertainty sets, i.e., A_k ($k = 1, 2$), and for set A_k at most k facilities will fail simultaneously. We change the performance bound on each uncertainty set and investigate the results. The detailed experimental setup is as follows: for Cases 1–4 (see Table 5), the performance bound on A_2 is always satisfied, and we change the bound on A_1 gradually; after we have obtained the location decision, we assume that one facility will be disrupted and compute the worst-case cost. Similarly, for Cases 5–10, the performance bound on A_1 is always satisfied, and we change the bound on A_2 and compute the normal and worst-case costs. The numerical analysis is conducted on instance 20%-10-20-30, and Table 5 and Figure 5 give the results.



(a) Cases 1–3 (b) Cases 5–9

Figure 5: Results of risk-constrained two-stage RO model

Table 5: Results for risk-constrained RO model for instance 20%-10-20-30

Case	ξ_1	ξ_2	Normal cost	FacN.	Worst-case cost	Time
1	**1100000**	1600000	758897	7	1103177	0.20
2	**1000000**	1600000	761983	8	976794	0.39
3	**800000**	1600000	766515	9	879850	0.78
4	**700000**	1600000	Infeasible[a]	N/A[b]	N/A	0.21
5	1100000	**1600000**	758897	7	1556769	0.36
6	1100000	**1400000**	761983	8	1366854	0.55
7	1100000	**1200000**	766515	9	1189425	1.26
8	1100000	**1000000**	771935	10	1045396	2.15
9	1100000	**800000**	794881	13	891019	3.24
10	1100000	**600000**	Infeasible	N/A	N/A	0.22

[a] The constraint is too tight and no feasible solution is found.

[b] Not applicable.

Table 5 shows that when the bounds on the worst-case performance are loose (i.e., Cases 1 and 5), the problem reduces to a generic LNDP. As the bounds become more restrictive, solutions with higher normal costs and lower worst-case costs are obtained, and more facilities are opened to improve reliability. In Cases 4 and 10, the model becomes infeasible, which suggests that if greater reliability is required, the system needs to obtain extra facilities or increase the capacity of some facilities. Therefore, the risk-constrained two-stage RO model can be employed as a decision support tool for system expansion with reliability considerations.

Figure 5 indicates that a slight increase in the normal cost can lead to a significant decrease in the worst-case cost. In particular, from Case 1 to 3, the normal cost increases by only 1.00%; however, the worst-case cost reduces by 20.24%. From Case 5 to 9, the worst-case cost reduces by 42.76% with only a 4.74% increase in the normal cost. We conclude that compared with G-LNDP, the risk-constrained RO model is capable of improving reliability substantially with only a slight increase in the normal cost.

5.4.4 Summary of model comparison

Based on the numerical experiments in Sections 5.4.1 to 5.4.3, we draw the following conclusions: (1) the proposed CC&G algorithm is able to solve the three two-stage RO models to optimality in a reasonable time. (2) the two-stage RO models can improve system reliability with only a slight increase in the normal cost. Thus, all the three robust models can be applied to situations where the decision-makers want to design a reliable supply chain network but without precise probability information about risks. (3) the expanded RO model can better balance cost, risk, and computational time, compared to the SP model and the basic RO model. It can be used to situations where the decision-makers want to reflect their attitudes or experiences about risks. If the algorithm does not converge after a preset time limit for the expanded model, we can change to the basic RO model. (4) we can use the risk-constrained model when we care more about the system's normal cost while still want to control the worst-case cost to some extent.

5.5 Parameter analysis

This section analyzes the effects of parameters.

5.5.1 Budget of uncertainty and partial disruption

As mentioned earlier, the scope of the uncertainty set and partial disruptions will affect the system design and operation. However, to what extent they will affect the cost remains unknown. To investigate this, we explore changing the value of m and δ simultaneously for instance 20%-10-20-30 and the RO-LNDP$_0$ model. Figure 6 presents the results.

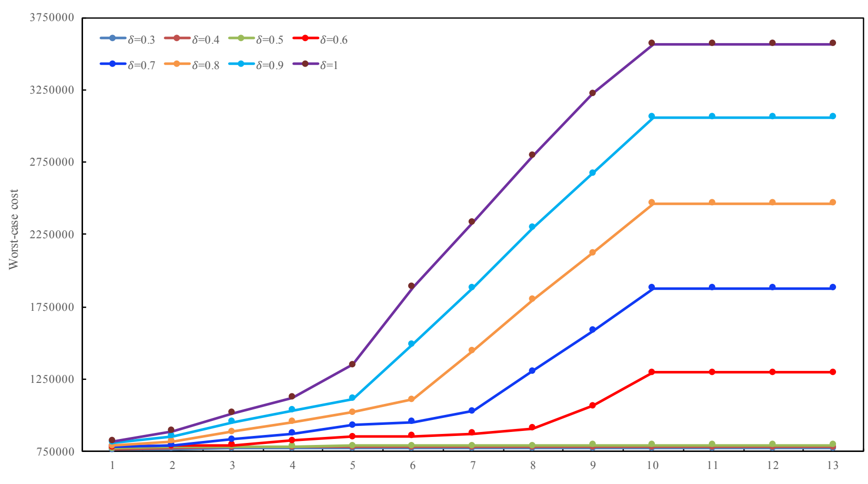


Figure 6: Impact of m and δ on cost for instance 20%-10-20-30

It can be seen that with both partial and complete disruption, the worst-case cost increases in general as the budget of the uncertainty set increases. For all values of δ, as m varies from 10 to 13, the cost remains stable. We explore the details of the solutions and find that in these cases the system does not open any facilities and all the demands are penalized. We also find that when partial disruption is considered, the cost for different values of m may be only slightly different. In particular, when δ is between 0.3 and 0.5, the variation is small although the budget of the uncertainty set increases gradually. This indicates that for regions where relatively minor disasters are likely, decision-makers can consider more disruptive scenarios with little increase in the worst-case cost. On the other hand, when δ is larger than 0.5, the system is much more sensitive to the budget of the uncertainty sets.

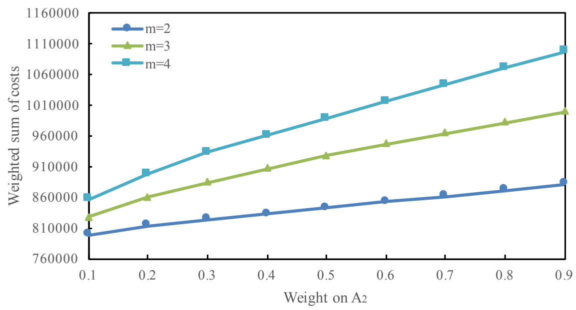
5.5.2 Weight of the uncertainty set

For the expanded RO model, the weights put on the uncertainty sets characterize decision-makers' protective level, which may influence the location decision. We conduct our analyses on four instances, where the number of facilities ranges from 30 to 60. For each instance, we consider two uncertainty sets A_1 and A_2: A_1 with $m_1 = 0$ (i.e., disruption-free case) and A_2 with $m_2 = 2$ or 3 or 4. We change the weight gradually and observe its influence. Results are presented in Figure 7, where the left side is the objective value of the RO-LNDP$_1$ model. The right side is the normal cost of the system, where we fix the location decision and solve a MCFP.

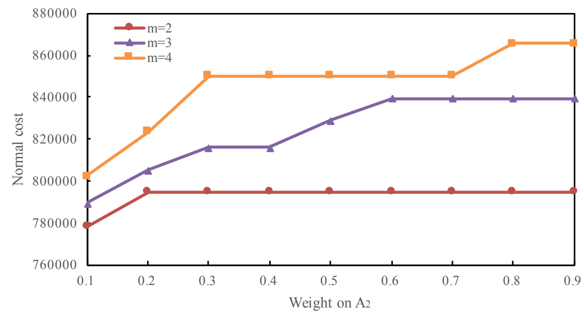
It shows that the objective value of the RO-LNDP$_1$ model increases almost linearly with ρ_2, which suggests that the worst-case performance of set A_2 accounts for a large portion of the objective value. When $m_2 = 2$, the normal cost is less sensitive to the value of ρ_2, especially for instance 30%-10-40-30. Normally there exists some regions where the normal cost keeps stable for each budget m_2. In these situations, the location decisions are the same. Therefore, it is possible that sometimes the estimating errors in the weight will not significantly influence the system's configuration. However, for the decision-makers, they should carefully determine the weight of larger uncertainty sets.

6 Conclusion

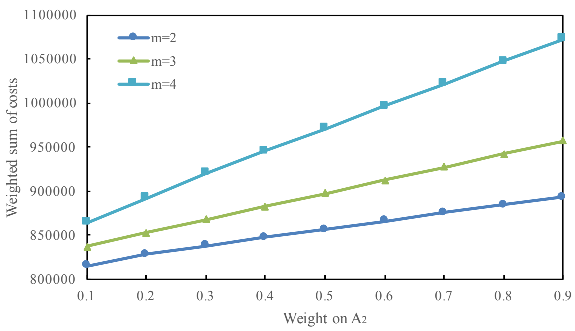
In this paper, we have presented a two-stage RO approach for the reliable LNDP. We use uncertainty sets to describe possible disruptive scenarios, and we have constructed three two-stage robust models. We use an exact algorithm, i.e., the C&CG algorithm, to solve these models to optimality. Our numerical tests show that (i) the C&CG algorithm, especially the C&CG LP method, outperforms BD in both solution quality and computational time; (ii) two-stage RO models give a considerable decrease in the cost of the worst disruptive situation for only a small increase in the normal cost; (iii) when



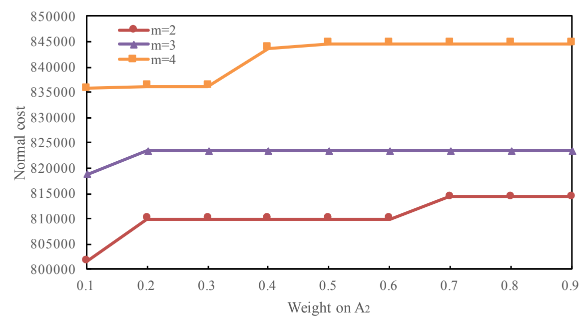
(a) Instance 20%-10-20-30



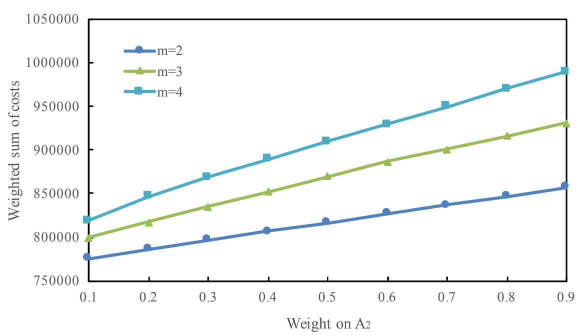
(b) Instance 20%-10-20-30



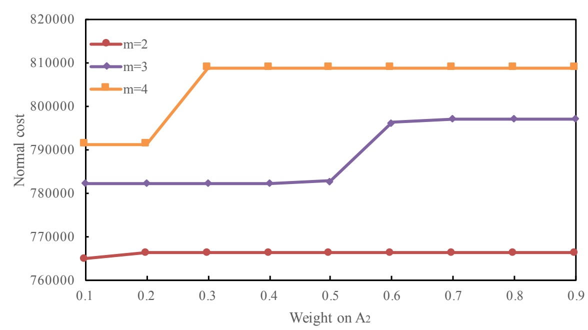
(c) Instance 30%-10-30-30



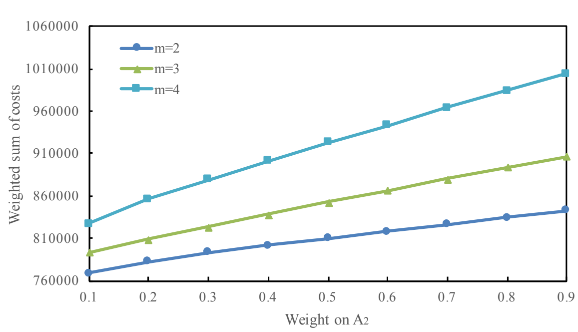
(d) Instance 30%-10-30-30



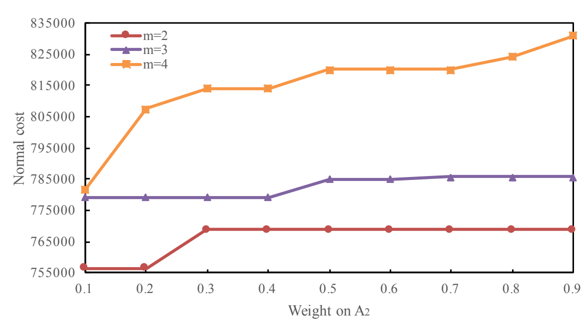
(e) Instance 30%-10-40-30



(f) Instance 30%-10-40-30



(g) Instance 30%-10-50-30



(h) Instance 30%-10-50-30

Figure 7: Impact of weight on the uncertainty set

partial disruption is considered, sometimes the system experiences small increases in the worst-case cost even when the scope of the uncertainty sets increases dramatically.

One possible extension of our work would be to consider the change in customer demand when disruptions occur, because the demand pattern may change considerably. It would also be interesting to combine reliable network design with other supply chain problems, such as vehicle routing and inventory management problems. Two-stage RO approach could also be applied to other optimization problems with an uncertainty component.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (71772100), Shenzhen Science and Technology Project (JCYJ20170412171044606), and National Key Technologies Research and Development Program (2016YFB0502601, 2016YFC0803107). We would like to thank anonymous referees for their helpful comments.

Appendix A Generic LNDP model

G-LNDP:

$$\min \sum_{j \in \mathcal{V}_0} f_j y_j + \sum_{(i,j) \in \mathcal{A}} c_{ij} x_{ij} + \sum_{j \in \mathcal{V}_\mathcal{D}} \theta_j u_j \tag{A1}$$

s.t.

$$\sum_{i \in \mathcal{V}_j^+} x_{ji} \leq b_j \quad \forall j \in \mathcal{V}_\mathcal{S} \tag{A2}$$

$$\sum_{i \in \mathcal{V}_j^+} x_{ji} = \sum_{i \in V_j^-} x_{ij} \quad \forall j \in \mathcal{V}_\mathcal{T} \tag{A3}$$

$$\sum_{i \in \mathcal{V}_j^-} x_{ij} + u_j = -b_j \quad \forall j \in \mathcal{V}_\mathcal{D} \tag{A4}$$

$$\sum_{i \in \mathcal{V}_j^+} x_{ji} \leq Q_j y_j \quad \forall j \in \mathcal{V}_0 \tag{A5}$$

$$y_j \in \{0, 1\} \quad \forall j \in \mathcal{V}_0 \tag{A6}$$

$$x_{ij} \geq 0 \quad \forall (i,j) \in \mathcal{A} \tag{A7}$$

$$u_j \geq 0 \quad \forall j \in \mathcal{V}_0 \tag{A8}$$

Appendix B Scenario-based stochastic programming model

Parameters and decision variables:

- S = set of disruptive scenarios
- p_s = occurrence probability of scenario $s \in S$
- x_{ijs} = product flow on arc (i, j) in scenario $s \in S$
- u_{js} = unsatisfied demand at node $j \in \mathcal{V}_D$ in scenario $s \in S$
- a_{js} = 1 if facility $j \in \mathcal{V}_0$ is disrupted in scenario $s \in S$, and 0 otherwise

$$\min \sum_{j \in \mathcal{V}_0} f_j y_j + \sum_{s \in S} p_s (\sum_{(i,j) \in \mathcal{A}} c_{ij} x_{ijs} + \sum_{j \in \mathcal{V}_\mathcal{D}} \theta_j u_{js}) \tag{B1}$$

s.t.

$$\sum_{i \in V_j^+} x_{jis} \leq b_j \quad \forall s \in S, j \in \mathcal{V}_\mathcal{S} \tag{B2}$$

$$\sum_{i \in V_j^+} x_{jis} = \sum_{i \in V_j^-} x_{ijs} \quad \forall s \in S, j \in \mathcal{V}_\mathcal{T} \tag{B3}$$

$$\sum_{i \in V_j^-} x_{ijs} + u_{js} = -b_j \quad \forall s \in S, j \in \mathcal{V}_\mathcal{D} \tag{B4}$$

$$\sum_{i \in V_j^+} x_{jis} \leq (1 - a_{js}) Q_j y_j \quad \forall s \in S, j \in \mathcal{V}_0 \tag{B5}$$

$$y_j \in \{0, 1\} \quad \forall j \in \mathcal{V}_0 \tag{B6}$$

$$a_{js} \in \{0, 1\} \quad \forall j \in \mathcal{V}_0, \forall s \in S \tag{B7}$$

$$x_{ijs} \geq 0 \quad \forall s \in S, (i, j) \in \mathcal{A} \tag{B8}$$

$$u_{js} \geq 0 \quad \forall s \in S, j \in \mathcal{V}_\mathcal{D} \tag{B9}$$

Note that although a_{js} is a binary variable in the model, we could allow it to be fractional, representing partial disruption.

References

Sibel A Alumur, Stefan Nickel, and Francisco Saldanha-da Gama. Hub location under uncertainty. *Transportation Research Part B: Methodological*, 46(4):529–543, 2012.

Yu An and Bo Zeng. Exploring the modeling capacity of two-stage robust optimization: Variants of robust unit commitment model. *IEEE Transactions on Power Systems*, 30(1):109–122, 2015.

Yu An, Yu Zhang, and Bo Zeng. The reliable hub-and-spoke design problem: Models and algorithms. *Optimization Online*, 2011.

Yu An, Bo Zeng, Yu Zhang, and Long Zhao. Reliable p-median facility location problem: two-stage robust models and algorithms. *Transportation Research Part B: Methodological*, 64:54–72, 2014.

Yu An, Yu Zhang, and Bo Zeng. The reliable hub-and-spoke design problem: Models and algorithms. *Transportation Research Part B: Methodological*, 77:103–122, 2015.

Alper Atamtürk and Muhong Zhang. Two-stage robust network flow and design under demand uncertainty. *Operations Research*, 55(4):662–673, 2007.

Nader Azad, Georgios KD Saharidis, Hamid Davoudpour, Hooman Malekly, and Seyed Alireza Yektamaram. Strategies for protecting supply chain networks against facility and transportation disruptions: an improved benders decomposition approach. *Annals of Operations Research*, 210(1):125–163, 2013.

Opher Baron, Joseph Milner, and Hussein Naseraldin. Facility location: A robust optimization approach. *Production and Operations Management*, 20(5):772–785, 2011.

Aharon Ben-Tal, Alexander Goryashko, Elana Guslitzer, and Arkadi Nemirovski. Adjustable robust solutions of uncertain linear programs. *Mathematical Programming*, 99(2):351–376, 2004.

Dimitris Bertsimas, David B Brown, and Constantine Caramanis. Theory and applications of robust optimization. *SIAM review*, 53(3):464–501, 2011.

Qi Chen, Xiaopeng Li, and Yanfeng Ouyang. Joint inventory-location problem under the risk of probabilistic facility disruptions. *Transportation Research Part B: Methodological*, 45(7):991–1003, 2011.

Ivan Contreras, Jean-François Cordeau, and Gilbert Laporte. The dynamic uncapacitated hub location problem. *Transportation Science*, 45(1):18–32, 2011.

Jean-François Cordeau, Federico Pasin, and Marius M Solomon. An integrated model for logistics network design. *Annals of operations research*, 144(1):59–82, 2006.

Tingting Cui, Yanfeng Ouyang, and Zuo-Jun Max Shen. Reliable facility location design under the risk of disruptions. *Operations Research*, 58(4-part-1):998–1011, 2010.

Zvi Drezner. Heuristic solution methods for two location problems with unreliable facilities. *Journal of the Operational Research Society*, 38(6):509–514, 1987.

Maryam Farahani, Hassan Shavandi, and Donya Rahmani. A location-inventory model considering a strategy to mitigate disruption risk in supply chain by substitutable products. *Computers & Industrial Engineering*, 108:213–224, 2017.

Virginie Gabrel, Mathieu Lacroix, Cécile Murat, and Nabila Remli. Robust location transportation problems under uncertain demands. *Discrete Applied Mathematics*, 164:100–111, 2014.

Ruiwei Jiang, Muhong Zhang, Guang Li, and Yongpei Guan. Benders' decomposition for the two-stage security constrained robust unit commitment problem. In *IIE Annual Conference. Proceedings*, page 1. Institute of Industrial Engineers-Publisher, 2012.

Xiaopeng Li and Yanfeng Ouyang. A continuum approximation approach to reliable facility location design under correlated probabilistic disruptions. *Transportation research part B: methodological*, 44(4):535–548, 2010.

Michael Lim, Mark S Daskin, Achal Bassamboo, and Sunil Chopra. A facility reliability problem: formulation, properties, and algorithm. *Naval Research Logistics (NRL)*, 57(1):58–70, 2010.

M Teresa Melo, Stefan Nickel, and Francisco Saldanha-Da-Gama. Facility location and supply chain management–a review. *European journal of operational research*, 196(2):401–412, 2009.

Hokey Min and Gengui Zhou. Supply chain modeling: past, present and future. *Computers & industrial engineering*, 43(1):231–249, 2002.

Stefan Mišković, Zorica Stanimirović, and Igor Grujičić. Solving the robust two-stage capacitated facility location problem with uncertain transportation costs. *Optimization Letters*, 11(6):1169–1184, 2017.

F Parvaresh, SM Moattar Husseini, SA Hashemi Golpayegany, and Behrooz Karimi. Hub network design problem in the presence of disruptions. *Journal of Intelligent Manufacturing*, 25(4):755–774, 2014.

Peng Peng, Lawrence V Snyder, Andrew Lim, and Zuli Liu. Reliable logistics networks design with facility disruptions. *Transportation Research Part B: Methodological*, 45(8):1190–1211, 2011.

Mir Saman Pishvaee, Reza Zanjirani Farahani, and Wout Dullaert. A memetic algorithm for bi-objective integrated forward/reverse logistics network design. *Computers & operations research*, 37(6):1100–1112, 2010.

Mir Saman Pishvaee, Masoud Rabbani, and Seyed Ali Torabi. A robust optimization approach to closed-loop supply chain network design under uncertainty. *Applied Mathematical Modelling*, 35(2):637–649, 2011.

Xuwei Qin, X Liu, and Lixin Tang. A two-stage stochastic mixed-integer program for the capacitated logistics fortification planning under accidental disruptions. *Computers & Industrial Engineering*, 65(4):614–623, 2013.

Farnaz Rayat, MirMohammad Musavi, and Ali Bozorgi-Amiri. Bi-objective reliable location-inventory-routing problem with partial backordering under disruption risks: A modified amosa approach. *Applied Soft Computing*, 59:622–643, 2017.

Shabnam Rezapour, Reza Zanjirani Farahani, and Morteza Pourakbar. Resilient supply chain network design under competition: A case study. *European Journal of Operational Research*, 259(3):1017–1035, 2017.

Tadeusz Sawik et al. *Supply chain disruption management using stochastic mixed integer programming*. Springer, 2018.

Zuo-Jun Max Shen, Roger Lezhou Zhan, and Jiawei Zhang. The reliable facility location problem: Formulations, heuristics, and approximation algorithms. *INFORMS Journal on Computing*, 23(3):470–482, 2011.

Davood Shishebori, Lawrence V Snyder, and Mohammad Saeed Jabalameli. A reliable budget-constrained fl/nd problem with unreliable facilities. *Networks and Spatial Economics*, 14(3-4):549–580, 2014.

Lawrence V Snyder and Mark S Daskin. Reliability models for facility location: the expected failure cost case. *Transportation Science*, 39(3):400–416, 2005.

Lawrence V Snyder and Mark S Daskin. Stochastic p-robust location problems. *IIE Transactions*, 38(11): 971–985, 2006.

Lawrence V Snyder, Maria P Scaparra, Mark S Daskin, and Richard L Church. Planning for disruptions in supply chain networks. *Tutorials in operations research*, 2:234–257, 2006.

Lawrence V Snyder, Zümbül Atan, Peng Peng, Ying Rong, Amanda J Schmitt, and Burcu Sinsoysal. Or/ms models for supply chain disruptions: A review. *IIE Transactions*, 48(2):89–109, 2016.

Ebrahim Teimuory, F Atoei, Emran Mohammadi, and A Amiri. A multi-objective reliable programming model for disruption in supply chain. *Management Science Letters*, 3(5):1467–1478, 2013.

Weijun Xie, Yanfeng Ouyang, and Sze Chun Wong. Reliable location-routing design under probabilistic facility disruptions. *Transportation Science*, 50(3):1128–1138, 2015.

Bo Zeng and Long Zhao. Solving two-stage robust optimization problems using a column-and-constraint generation method. *Operations Research Letters*, 41(5):457–461, 2013.

Ying Zhang, Mingyao Qi, Wei-Hua Lin, and Lixin Miao. A metaheuristic approach to the reliable location routing problem under disruptions. *Transportation Research Part E: Logistics and Transportation Review*, 83:90–110, 2015.

Ying Zhang, Lawrence V Snyder, Mingyao Qi, and Lixin Miao. A heterogeneous reliable location model with risk pooling under supply disruptions. *Transportation Research Part B: Methodological*, 83:151–178, 2016.

Long Zhao and Bo Zeng. Robust unit commitment problem with demand response and wind energy. In *Power and Energy Society General Meeting, 2012 IEEE*, pages 1–8. IEEE, 2012.