Online Supplement: Team Orienteering with Time-Varying Profit

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Abstract

This is the online supplement of the paper: Team Orienteering with Time-Varying Profit. Section 1 describes the details of components of the ILS precedure in the hybrid heuristic algorithm ILS-MCS. Section 2 presents the instance generation. Section 3 provides the detailed results of the computational experiments discussed in Section 6 of the main article.

1 Detailed Description of the ILS-MCS algorithm

The ILS procedure discussed in Section 5 of the article consists of three main phases, i.e., initialization, perturbation and local search.

1.1 Initialization Phase

Considering that more than one factors have an impact on collecting profit, it is difficult to deal with the visiting priority under these interactive factors. For example, a customer with a large profit may attract the visit. However, if it requires a long service time, delaying the visit to succeeding customers may result in higher loss of profit. In this case, it is possible that prioritizing a visit to a nearer customer with a larger delay ratio may not be as profitable as a visit to a further one with a smaller delay ratio. To take into account such different contributing factors, we assign a score to each customer. More specifically, a customer with a larger ideal profit, smaller delay rate, steeper collecting coefficient and shorter travel time required to traverse will have a higher priority to be visited first. We compute the score of customer i as follows:

$$\zeta_{iv} = \frac{p_i \beta_i}{d_i \Delta t_{iv}} \tag{1}$$

where p_i, d_i and β_i are ideal profit, delay rate and profit collection rate, respectively. Δt_{iv} is a route-specific value which equals the incremental travel time when customer i is added to the tail of route k. Let r_v denote the last customer visited by vehicle v, we have $\Delta t_{iv} = t_{r_v,i} + t_{i,n+1} - t_{r_v,n+1}$.

To construct the initial solution, we employ a parallel insertion method which builds $|V|$ routes simultaneously. Firstly, $|V|$ routes that only consist of the starting depot 0 and ending depot $n+1$ are initialized. Let τ_v denote the departure time from the last customer r_v in route R_v and $\hat{V}$ denote a set of incomplete routes. Then, we try to assign the customer i with the highest score ζ_{iv} to the associating route R_v. If the insertion does not satisfy the time limit constraint even without considering the service time of customer i, the associating route R_v is reviewed as a complete route and removed from the incomplete set $\hat{V}$. Otherwise, the insertion procedure is called. Recall that service time must be spent to obtain profit, thus, it is plausible to assume that the service time spent on the visiting customer is larger than zero. Thus, we set the amount of service time s_i at each customer i is as long as to collect 50% of profit when the remaining time is sufficient, otherwise, the amount of service time is the remaining time. We provide the pseudo-code in Algorithm 1.

1.2 Perturbation Phase

The perturbation method is a key component in ILS where a perturbation is carried out on the local optimal solution. The main idea is to remove some visited customers and then insert some unvisited customers to generate a diversified solution.

We define a perturbation strength parameter π to indicate the number of customers to be removed from the routes. The range of π belongs to $[\pi_{\min}, \pi_{\max}] = [|V|/2, |N(sol)|/2]$. Initially, we set $\pi = \pi_{\min}$ and increase its value by 1 if no better solution is found. Once a new best solution is found, π is reset to $\pi_{\min}$. This way, the search focuses on the promising

Algorithm 1 Initialization

Input: the set of customers $\hat{N} = \{1, \dots, n\}$, the decision horizon $T_{\max}$, the number of vehicles $|V|$
Let $sol = \{(R^v, S^v) | R^v = \{0, n+1\}, S^v = \{0, 0\}, \forall v \in V\}$
Set $r_v \leftarrow 0$, $\tau_v \leftarrow 0$ for every route $v \in V$ and $\hat{V} = \{1, \dots, |V|\}$
while $\hat{V} \neq \emptyset$ **do**
  for all $i \in \hat{N}$ **do**
    for all $v \in V$ **do**
      $\zeta_{iv} = \frac{p_i \beta_i}{d_i \Delta t_{iv}}$.
    end for
  end for
  Choose $(i^*, v^*) \leftarrow \arg\max_{i \in \hat{N}, v \in V} \{\zeta_{iv}\}$
  if $\tau_{v^*} + t_{r_{v^*}, i^*} + t_{i^*, (n+1)} \leq T_{\max}$ **then**
    $s_{i^*} = \min(s_i, T_{\max} - (\tau_{v^*} + t_{r_{v^*}, i^*} + t_{i^*, (n+1)}))$
    Update $\tau_{v^*} \leftarrow \tau_{v^*} + t_{r_{v^*}, i^*} + s_{i^*}$, $r_{v^*} \leftarrow i^*$
    $R^{v^*} = \{0, \dots, i^*, n+1\}$, $S^{v^*} = \{0, \dots, s_{i^*}, 0\}$, $\hat{N} \leftarrow \hat{N} \setminus i^*$
  else
    $\hat{V} \leftarrow \hat{V} \setminus v^*$
  end if
end while
Output: sol

region around the local optima during the early stage while it moves to the unexplored region further away from the local optima at a later stage. Once the number of customers to be removed is determined, we then randomly select π customers and remove them from the local optimal solution.

To diversify the search to unexplored region, we insert some unvisited customers into the current solution after removal. First, to define an unexplored region, we assign a number to each customer to record the times it has been inserted into the local optimal solution successfully in the past perturbation phase. Then we sort all unvisited customers in ascending order of this number, and the customers at the top of the list are more likely to be in the unexplored region. Therefore, the algorithm proceeds to check these customers successively. For each customer, it attempts to find all inserting positions satisfying all constraints. Note that the amount of service time of this inserting customer is supposed to be zero. Then, it randomly chooses a feasible position to insert this customer. If no such position can be found, then it turns to the next customer. The insertion will continue until all customers in the unexplored region have been checked or the total travel time of the current route beyond half of $T_{\max}$.

1.3 Local Search Phase

Local search is another important part in ILS, here we define eight neighbourhood structures based on the classical neighborhoods in routing problem and the characteristic of our problem. A given solution can be improved by adjusting the order of customers in a route to reduce the travel time, by replacing the position of a customer in order to reduce the delay penalty, or by removing some customers increase the service time spent at other customers. We choose some classical operators for the routing problem and also design additional neighborhood search operators. They are presented as follows:

- *Or-opt*: Relocate a chain of σ consecutive customers in the same route, we set $\sigma = 1, 2, 3$ here.
- *swap*: Swap the position of two customers in the same route.
- *2-opt*: Remove two non-adjacent edges from the route and replace with other two edges, we notice that the sequence between these two removed edges are reversed.
- λ-*Exchange*: Change the locations of two chains of λ consecutive customers located in a pair of routes (r_i, r_j), we set $\lambda \leq \lceil \min\{|r_i|, |r_j|\}/2 \rceil$.
- *Cross*: Remove two edges from two routes and replace them with other two edges.
- *Removal*: Remove all visited but unserved customers from the solution to extend the service time.
- *Insertion*: Insert an unvisited customer into the solution to increase the profit.
- *Replacement*: Replace a visited customer v_i by an unvisited customer v_j with the possibility $\min\{1, \frac{p_j}{p_i}\}$

The first three neighborhood search operators are intra-route operators. These operators aim to improve the profit of a route by adjusting the order of visiting customers. The following two are inter-route operators. They are used to balance the whole profit of a solution via changing customers to different routes. While the last three operators are designed to change the set of visited customers. The removal operator will remove unserved customers from every route in the solution since we know that traversing a customer without any service is time-wasting since no return will be obtained. Thus, the solution will be improved if there exist an unserved customer. After removing unserved customers, the solution may have a place to accommodate other unvisited customers, thus insertion is called following the removal operator. We also attempt to improve the current solution by replacing visited customers by unvisited customers. In each iteration, we implement these eight neighborhoods in the given sequence, and for every neighborhood, we use the best improvement strategy and return the best improved neighborhood if any.

1.3.1 Rules to Reduce the Neighbourhood Size.

In a neighbourhood operator, many neighbours of the current solution or route are generated. Consider the complete evaluation of a solution is time-consuming, we set two rules to discard some uninteresting neighbours as follows. Let sol' denotes a neighbour generated from solution sol using the move operator.

Rule 1: Given the solution sol, in which the service times of all customers are known, the total collected profit $f(sol)$ can be calculated exactly using the objective function. If we assume that the profit at each customer can be immediately obtained when the vehicle arrives at the customer without spending any amount of service time. Then an upper bound of the total profit $f_{UB}(sol')$ of the neighbour sol' can be calculated as $\sum_{v_i \in N_{sol'}} (p_{v_i} - d_{v_i} a_{v_i})$, where $N_{sol'}$ is the set of all visited customers by solution sol' and a_{v_i} is the arrival time at visited customer v_i without any service time. Then we define the neighbour solution sol' is an uninteresting neighbour if $f_{UB}(sol') \leq f(sol)$, as it is obvious that this neighbour solution cannot result in a higher profit than the current one.

Rule 2: As the service time take a crucial role in collecting the profit, we consider sol' as an uninteresting neighbour if the amount of total available service time $\hat{T}'$ left in the neighbour solution sol' is less than 50% of the amount $\hat{T}$ in the current solution sol.

2 Details of the Instances

We generate the instances based on the Solomon instances[1]. Specifically, we consider the R(C) type network extracting from R101(C101) data file with first 25 customers. In each

[1]The Solomon instances are obtained from http://neo.lcc.uma.es/vrp/vrp-instances/capacitated-vrp-with-time-windows-instances/.

instance, the parameters, i.e., x coordinate x_i, y coordinate y_i and demand p_i (the ideal profit in our problem) are taken from Solomon's instances. We take the first node to be the starting and ending depot, and we compute t_{ij} using the Euclidean distance and round it to the nearest hundredth. The other parameters in our problem are conducted from the location (x_i, y_i) using a similar generation scheme as in Erdoğan and Laporte (2013). More specifically, let $X_{\max}, X_{\min}, Y_{\max}$ and $Y_{\min}$ denote the maximum X coordinate, minimum X coordinate, maximum Y coordinate and minimum Y coordinate of all customers in the instance, respectively. Then we determine a time factor as $L = 2\lfloor X_{\max} - X_{\min} + Y_{\max} - Y_{\min} \rfloor$. Next we set two factors as $r_{\min} = 0.2$ and $r_{\max} = 0.5$ and determine the delay rate as $d_i = (r_{\min} + ((\lfloor X_i + Y_i \rfloor \bmod 30)/30) * (r_{\max} - r_{\min}))/25$. The collecting coefficient β_i is also determined in the similar manner, using the formula $\beta_i = (r_{\min} + ((\lfloor X_i + Y_i \rfloor \bmod 20)/20) * (r_{\max} - r_{\min}))/20$.

For each type of network, we generate a total of 48 instances. Each instance is characterized by the number of customers n, the number of vehicles V and the length of decision horizon $T_{\max}$: The number of customers n belongs to $\{6, 9, 25\}$. The number of vehicles V belongs to $\{2, 3, 4, 5\}$ and the length of time limit equal to $T_{\max} \in \{L, 2L, 3L, 4L\}$. Here we notice that the 6(9) customers are the first 6(9) among 25 customers in R type network, while for the C type network, the 6(9) customers are chosen from 2(3) clusters.

We note here that this instance generation reflects the realistic situation in the context of humanitarian search and rescue operation in the urban areas. First, we choose R type and C type networks to represent different configurations of the clusters because the affected sites could be located far from each other or could be clustered in certain specific regions. Secondly, a separate profit function is designed for each location. In other words, the parameters (p_i, d_i and β_i) take different values for different locations. This reflects the different numbers of people to be rescued, the different damage levels and structure types of building at each affected site. Finally, the values of β_i generated in this way generally make the service times to be longer than the travel times between locations, which reflect the situation in SAR, in particular in the urban areas. For example, it would take much longer time to work in a collapsed building than the time it takes to arrive there.

3 Detailed Computational Results

The detailed results on the exact method BBC and its enhancement are presented in Section 3.1. Section 3.2 describes the route generation for the service time scheduling problem and the parameters using in the different NLP methods. Section 3.3 provides the detailed results on the ILS-MCS. Finally, a discussion on the modified objective function is offered in Section 3.4, together with numerical experiments and results. We mention here that the detailed results of all the instances can be also obtained from `https://github.com/QinxiaoYu/TOPTTP`.

3.1 Detailed Results on the Exact Methods BBC and Enhanced BBC

Tables 1-3 present the performance of the SCIP, the BBC and the enhanced BBC for the instances with 6, 9 and 25 customers. We highlight the best found objective value of each instances using bold font in Tables 2 and 3.

3.2 Experimental Settings for the Comparisons of the Algorithms for the Service Time Scheduling Subproblem

We demonstrate the effectiveness of the MCS by comparing it with other NLP methods on a predetermined group of routes. The number of customers in a route varies from 2 to 30, and five routes are conducted randomly for each number of customers. We use C101 data file which includes 100 customers and 1 depot to generate these routes and define $T_{\max} = 4L$ for all routes, where L is perimeter of the minimum bounding rectangle of the 101 nodes.

We performed preliminary experiments for hyper-parameter tuning for all the algorithms to determine the best-performing ones. The following parameters are used in the MCS: $\alpha = 10$, $\alpha_{\min} = 1$ and $||\vec{e_i}|| = 0.01$. For the IpOpt solver, we use the default settings (as it generally gives the best results) where the IpOpt solver terminates when the NLP optimality error becomes smaller than 10^{-3}. For the MMA, SLSQP, COBYLA and ISRES methods available from Johnson (2019), we set the relative tolerance on function value as 10^{-7} and the maximum wall-clock time as 1 second, so the algorithm stops before 1 second if the increase in the function value from one iteration to next relative to the function value is less than 10^{-7}. For the NOMAD and CS methods available in Abramson et al. (2019), we set the maximum wall-clock time as 1 second, the direction type as *OrthoMADS_2N* for the NOMAD method and *GPS_2N_STATIC* for the CS method, we refer readers to Audet et al. (2009) for more details about these parameters.

Table 1: Results Comparing Three Exact Methods on Instances with 6 Customers

Instance	SCIP		BBC		Enhanced BBC	
	z^*	Time(s)	z^*	Time(s)	z^*	Time(s)
C6/70/2	15.88	34.2	15.88	25.9	15.88	0.4
C6/140/2	30.25	79.1	30.25	47.7	30.25	2.0
C6/210/2	40.90	225.4	40.90	60.8	40.90	4.7
C6/280/2	48.01	453.5	48.01	53.8	48.01	10.2
C6/70/3	20.66	25.6	20.66	13.0	20.66	0.9
C6/140/3	40.91	50.5	40.91	18.4	40.91	1.0
C6/210/3	52.74	68.5	52.74	22.8	52.74	3.4
C6/280/3	59.52	89.6	59.52	20.0	59.52	2.2
C6/70/4	25.25	11.1	25.25	6.5	25.25	0.5
C6/140/4	48.52	10.1	48.52	6.9	48.52	0.9
C6/210/4	60.83	17.7	60.83	5.1	60.83	1.1
C6/280/4	66.82	18.2	66.82	4.9	66.82	1.1
C6/70/5	29.28	5.8	29.28	1.4	29.28	0.2
C6/140/5	54.99	2.8	54.99	1.4	54.99	0.3
C6/210/5	66.57	2.7	66.57	1.9	66.57	0.3
C6/280/5	71.71	1.8	71.71	1.6	71.71	0.4
Avg	*45.80*	*68.5*	*45.80*	*18.3*	*45.80*	*1.9*
R6/144/2	35.67	51.0	35.67	28.4	35.67	0.3
R6/288/2	52.98	121.2	52.98	52.1	52.98	3.4
R6/432/2	60.98	173.2	60.98	60.2	60.98	7.8
R6/576/2	63.70	155.6	63.70	53.9	63.70	14.1
R6/144/3	45.58	43.1	45.58	19.1	45.58	0.5
R6/288/3	63.83	42.5	63.83	20.3	63.83	2.0
R6/432/3	68.66	50.8	68.66	24.9	68.66	4.0
R6/576/3	69.85	47.2	69.85	21.4	69.85	5.5
R6/144/4	52.21	14.5	52.21	4.6	52.21	0.6
R6/288/4	69.09	10.0	69.09	4.7	69.09	1.2
R6/432/4	72.29	12.7	72.29	4.5	72.29	1.9
R6/576/4	72.85	15.4	72.85	4.9	72.85	2.7
R6/144/5	58.05	4.6	58.05	1.1	58.05	0.5
R6/288/5	72.50	4.9	72.50	1.0	72.50	0.7
R6/432/5	74.72	4.7	74.72	1.0	74.72	0.8
R6/576/5	75.08	4.5	75.08	1.2	75.08	1.2
Avg	*63.00*	*47.2*	*63.00*	*19.0*	*63.00*	*2.9*

Table 2: Results Comparing Three Exact Methods on Instances with 9 Customers

Instance	SCIP				BBC				Enhanced BBC			
	z_{lb}	z_{ub}	Gap	Time(s)	z_{lb}	z_{ub}	Gap	Time(s)	z_{lb}	z_{ub}	Gap	Time(s)
C9/116/2	35.69	87.03	143.9%	7200.0	**36.32**	88.69	144.2%	7200.0	**36.32**	36.32	0.0%	2.9
C9/232/2	62.29	128.27	105.9%	7200.0	62.98	128.27	103.7%	7200.0	**63.68**	63.69	0.0%	196.2
C9/348/2	80.32	133.73	66.5%	7200.0	80.25	135.09	68.3%	7200.0	**80.57**	85.60	6.2%	7200.0
C9/464/2	89.48	135.92	51.9%	7200.0	90.35	136.54	51.1%	7200.0	**90.53**	109.26	20.7%	7200.0
C9/116/3	46.28	86.98	87.9%	7200.0	45.73	88.69	94.0%	7200.0	**46.47**	46.47	0.0%	6.9
C9/232/3	77.69	128.27	65.1%	7200.0	80.73	128.27	58.9%	7200.0	**81.46**	81.47	0.0%	862.7
C9/348/3	96.00	135.09	40.7%	7200.0	98.34	135.09	37.4%	7200.0	**98.78**	101.73	3.0%	7200.0
C9/464/3	106.48	136.54	28.2%	7200.0	106.63	136.54	28.0%	7200.0	**106.82**	127.48	19.3%	7200.0
C9/116/4	**56.44**	56.44	0.0%	5990.7	56.40	88.65	57.2%	7200.0	**56.44**	56.44	0.0%	15.1
C9/232/4	94.26	126.24	33.9%	7200.0	95.06	128.27	34.9%	7200.0	**95.26**	95.27	0.0%	565.9
C9/348/4	110.36	134.16	21.6%	7200.0	110.70	135.09	22.0%	7200.0	**110.96**	110.97	0.0%	1647.5
C9/464/4	106.23	136.54	28.5%	7200.0	116.85	136.54	16.9%	7200.0	**116.98**	116.99	0.0%	6840.9
C9/116/5	**66.17**	66.17	0.0%	2138.5	**66.17**	66.18	0.0%	3807.7	**66.17**	66.18	0.0%	24.0
C9/232/5	**105.12**	121.45	15.5%	7200.0	**105.21**	105.21	0.0%	4792.3	**105.21**	105.22	0.0%	381.0
C9/348/5	117.78	134.17	13.9%	7200.0	**118.94**	118.95	0.0%	4600.5	**118.94**	118.95	0.0%	1117.9
C9/464/5	117.25	136.54	16.5%	7200.0	**123.00**	123.01	0.0%	3882.4	**123.00**	123.01	0.0%	1601.0
Avg	*85.49*	*117.72*	*45.0%*	*6808.08*	*87.10*	*117.44*	*44.8%*	*6467.7*	*87.35*	*90.31*	*3.1%*	*2628.9*
R9/176/2	40.23	87.53	117.6%	7200.0	39.72	92.54	133.0%	7200.0	**40.39**	40.39	0.0%	5.2
R9/352/2	61.99	102.94	66.0%	7200.0	64.28	104.10	62.0%	7200.0	**64.92**	64.92	0.0%	2451.3
R9/528/2	70.28	105.05	49.5%	7200.0	72.99	105.05	43.9%	7200.0	**73.99**	87.73	18.6%	7200.0
R9/704/2	76.13	105.00	37.9%	7200.0	75.80	105.17	38.7%	7200.0	**76.71**	93.19	21.5%	7200.0
R9/176/3	**56.12**	90.50	61.3%	7200.0	55.63	92.54	66.3%	7200.0	**56.12**	56.12	0.0%	12.6
R9/352/3	79.23	103.32	30.4%	7200.0	**80.09**	104.10	30.0%	7200.0	**80.09**	80.10	0.0%	3517.1
R9/528/3	81.14	105.05	29.5%	7200.0	84.31	105.05	24.6%	7200.0	**86.45**	93.63	8.3%	7200.0
R9/704/3	85.83	105.17	22.5%	7200.0	**87.47**	105.17	20.2%	7200.0	**87.47**	98.33	12.4%	7200.0
R9/176/4	**65.99**	65.99	0.0%	2682.7	65.62	92.54	41.0%	7200.0	**65.99**	65.99	0.0%	166.7
R9/352/4	**88.46**	102.66	16.1%	7200.0	88.12	104.10	18.1%	7200.0	**88.46**	88.47	0.0%	1797.8
R9/528/4	91.69	104.76	14.3%	7200.0	91.72	105.05	14.5%	7200.0	**92.57**	92.58	0.0%	2832.4
R9/704/4	92.19	105.17	14.1%	7200.0	**93.20**	105.17	12.8%	7200.0	**93.20**	93.21	0.0%	5808.0
R9/176/5	**74.14**	74.14	0.0%	3377.5	**74.14**	74.15	0.0%	4677.4	**74.14**	74.15	0.0%	74.8
R9/352/5	88.46	102.66	16.1%	7200.0	**93.92**	93.93	0.0%	4224.3	**93.92**	93.93	0.0%	560.3
R9/528/5	**96.29**	102.67	6.6%	7200.0	**96.29**	96.30	0.0%	3923.0	**96.29**	96.30	0.0%	895.5
R9/704/5	96.39	105.15	9.1%	7200.0	**96.55**	96.56	0.0%	3972.1	**96.55**	96.56	0.0%	1243.6
Avg	*77.79*	*97.99*	*30.7%*	*6678.76*	*78.74*	*98.84*	*31.6%*	*6449.8*	*79.20*	*82.23*	*3.8%*	*3010.4*

Table 3: Results Comparing Three Exact Methods on Instances with 25 Customers

Instance	SCIP				BBC				Enhanced BBC			
	z_{lb}	z_{ub}	Gap	Time(s)	z_{lb}	z_{ub}	Gap	Time(s)	z_{lb}	z_{ub}	Gap	Time(s)
C25/130/2	60.92	317.85	421.71%	7200.0	**62.03**	317.85	412.4%	7200.0	59.34	74.02	24.7%	7200.0
C25/260/2	95.72	430.34	349.59%	7200.0	102.17	430.33	321.2%	7200.0	**122.40**	145.37	18.8%	7200.0
C25/390/2	153.47	447.92	191.87%	7200.0	120.11	447.92	272.9%	7200.0	**168.38**	224.03	33.0%	7200.0
C25/520/2	180.64	451.31	149.84%	7200.0	127.45	451.31	254.1%	7200.0	**199.68**	236.35	18.4%	7200.0
C25/130/3	71.12	317.85	346.91%	7200.0	**87.43**	317.85	263.5%	7200.0	85.33	109.74	28.6%	7200.0
C25/260/3	134.39	430.33	220.20%	7200.0	137.95	430.33	211.9%	7200.0	**167.26**	195.53	16.9%	7200.0
C25/390/3	189.96	447.92	135.80%	7200.0	159.39	447.92	181.0%	7200.0	**215.61**	252.67	17.2%	7200.0
C25/520/3	227.10	451.30	98.72%	7200.0	167.63	451.31	169.2%	7200.0	**250.22**	296.36	18.4%	7200.0
C25/130/4	87.29	317.85	264.14%	7200.0	**102.97**	317.85	208.7%	7200.0	102.68	139.19	35.6%	7200.0
C25/260/4	167.85	430.34	156.38%	7200.0	170.11	430.33	153.0%	7200.0	**192.78**	234.26	21.5%	7200.0
C25/390/4	213.86	447.92	109.44%	7200.0	196.64	447.92	127.8%	7200.0	**246.15**	366.80	49.0%	7200.0
C25/520/4	255.27	451.32	76.80%	7200.0	206.55	451.31	118.5%	7200.0	**270.04**	429.75	59.1%	7200.0
C25/130/5	97.86	317.85	224.80%	7200.0	115.52	317.85	175.2%	7200.0	**119.61**	162.42	35.8%	7200.0
C25/260/5	189.25	430.33	127.39%	7200.0	192.01	430.33	124.1%	7200.0	**224.62**	266.59	18.7%	7200.0
C25/390/5	245.91	447.92	82.15%	7200.0	221.29	447.92	102.4%	7200.0	**271.47**	422.02	55.5%	7200.0
C25/520/5	283.68	451.30	59.09%	7200.0	232.13	451.31	94.4%	7200.0	**303.75**	451.31	48.6%	7200.0
Avg	*165.89*	*411.85*	*188.43%*	*7200.0*	*150.08*	*411.85*	*199.4%*	*7200.0*	*187.46*	*250.40*	*31.2%*	*7200.0*
R25/240/2	32.59	303.60	831.47%	7200.0	64.72	303.60	369.1%	7200.0	**77.12**	79.59	3.2%	7200.0
R25/480/2	82.53	322.40	290.67%	7200.0	90.09	322.40	257.9%	7200.0	**124.86**	150.88	20.8%	7200.0
R25/720/2	104.75	323.41	208.74%	7200.0	94.14	323.41	243.5%	7200.0	**145.95**	199.02	36.4%	7200.0
R25/960/2	94.48	323.48	242.39%	7200.0	94.66	323.48	241.7%	7200.0	**158.29**	234.43	48.1%	7200.0
R25/240/3	73.00	303.61	315.90%	7200.0	91.54	303.60	231.6%	7200.0	**103.68**	115.36	11.3%	7200.0
R25/480/3	131.31	322.40	145.53%	7200.0	141.80	322.40	127.4%	7200.0	**164.02**	197.97	20.7%	7200.0
R25/720/3	145.26	323.41	122.64%	7200.0	154.08	323.41	109.9%	7200.0	**170.60**	298.91	75.2%	7200.0
R25/960/3	144.33	323.49	124.13%	7200.0	156.86	323.48	106.2%	7200.0	**172.43**	323.48	87.6%	7200.0
R25/240/4	81.00	303.60	274.82%	7200.0	120.92	303.60	151.1%	7200.0	**126.02**	150.89	19.7%	7200.0
R25/480/4	161.19	322.39	100.01%	7200.0	**177.98**	322.40	81.1%	7200.0	175.47	297.17	69.4%	7200.0
R25/720/4	188.53	323.40	71.54%	7200.0	191.83	323.41	68.6%	7200.0	**201.27**	323.41	60.7%	7200.0
R25/960/4	98.41	323.48	228.70%	7200.0	**194.70**	323.48	66.1%	7200.0	193.57	323.48	67.1%	7200.0
R25/240/5	102.51	303.60	196.16%	7200.0	139.49	303.60	117.6%	7200.0	**143.08**	214.78	50.1%	7200.0
R25/480/5	157.68	322.40	104.47%	7200.0	**205.82**	322.40	56.6%	7200.0	197.44	319.40	61.8%	7200.0
R25/720/5	194.04	323.40	66.67%	7200.0	**221.34**	323.41	46.1%	7200.0	219.05	323.41	47.6%	7200.0
R25/960/5	-	-	-	7200.0	**224.30**	323.48	44.2%	7200.0	224.28	323.48	44.2%	7200.0
Avg	*119.44*	*317.87*	*221.59%*	*7200.0*	*142.66*	*317.87*	*151.6%*	*7200.0*	*158.19*	*236.81*	*45.3%*	*7200.0*

- indicates that SCIP did not find a solution within 2 hours.

3.3 Detailed Results of the Heuristic Method ILS-MCS

3.3.1 Detailed Results of ILS-MCS and ILS-IpOpt on Small Instances (6, 9 and 25 Customers)

The results of heuristic methods ILS-MCS and ILS-IpOpt on instances with 6, 9 and 25 customers are shown in Tables 4-6, we also list the results of the enhanced BBC in these three tables for easy comparison. Boldface letters are used to indicate the best found objective value of each instances.

3.3.2 Detailed Results of ILS-MCS on Large Instances (50, 75 and 100 Customers)

In Tables 7-9, we detail the results from the enhanced BBC, ILS-IpOpt and ILS-MCS for instances with 50, 75 and 100 customers. Boldface letters are used to indicate the best found objective value of each instances.

Table 4: Results of the Heuristic Methods on Instance with 6 Customers

Instance	Enhanced BBC				ILS-IpOpt						ILS-MCS					
	z_{lb}	z_{ub}	Gap	Found Time(s)	Best Obj	Worst Obj	Avg. Obj	Std. Dev.	Gap	Avg. Time(s)	Best Obj	Worst Obj	Avg. Obj	Std. Dev.	Gap	Avg. Time(s)
C6/70/2	**15.88**	15.88	0.0%	0.36	**15.88**	15.88	15.88	0.00	0.0%	0.98	**15.88**	15.88	15.88	0.00	0.0%	0.03
C6/140/2	**30.25**	30.25	0.0%	1.77	**30.25**	30.25	30.25	0.00	0.0%	3.07	**30.25**	30.25	30.25	0.00	0.0%	0.05
C6/210/2	**40.90**	40.90	0.0%	2.47	**40.90**	40.90	40.90	0.00	0.0%	2.94	**40.90**	40.90	40.90	0.00	0.0%	0.06
C6/280/2	**48.01**	48.01	0.0%	1.43	**48.01**	48.01	48.01	0.00	0.0%	3.99	**48.01**	48.01	48.01	0.00	0.0%	0.09
C6/70/3	**20.66**	20.66	0.0%	0.85	**20.65**	20.65	20.65	0.00	0.0%	1.67	**20.65**	20.65	20.65	0.00	0.0%	0.05
C6/140/3	**40.91**	40.91	0.0%	0.36	**40.91**	40.91	40.91	0.00	0.0%	1.67	**40.91**	40.91	40.91	0.00	0.0%	0.06
C6/210/3	**52.74**	52.74	0.0%	2.85	**52.74**	52.74	52.74	0.00	0.0%	1.71	**52.74**	52.74	52.74	0.00	0.0%	0.06
C6/280/3	**59.52**	59.52	0.0%	0.79	**59.52**	59.52	59.52	0.00	0.0%	1.78	**59.52**	59.52	59.52	0.00	0.0%	0.06
C6/70/4	**25.25**	25.25	0.0%	0.44	**25.25**	25.25	25.25	0.00	0.0%	0.60	**25.25**	25.25	25.25	0.00	0.0%	0.04
C6/140/4	**48.52**	48.52	0.0%	0.72	**48.52**	48.52	48.52	0.00	0.0%	1.01	**48.52**	48.52	48.52	0.00	0.0%	0.08
C6/210/4	**60.83**	60.83	0.0%	0.94	**60.83**	60.83	60.83	0.00	0.0%	1.00	**60.83**	60.83	60.83	0.00	0.0%	0.06
C6/280/4	**66.82**	66.82	0.0%	0.81	**66.82**	66.75	66.79	0.03	0.0%	0.97	**66.82**	66.82	66.82	0.00	0.0%	0.05
C6/70/5	**29.28**	29.28	0.0%	0.13	**29.28**	29.28	29.28	0.00	0.0%	0.54	**29.28**	29.28	29.28	0.00	0.0%	0.03
C6/140/5	**54.99**	54.99	0.0%	0.28	**54.93**	54.93	54.93	0.00	0.1%	0.45	**54.93**	54.93	54.93	0.00	0.1%	0.03
C6/210/5	**66.57**	66.57	0.0%	0.13	**66.57**	66.57	66.57	0.00	0.0%	0.52	**66.57**	66.57	66.57	0.00	0.0%	0.04
C6/280/5	**71.71**	71.71	0.0%	0.07	**71.71**	71.71	71.71	0.00	0.0%	0.48	**71.71**	71.71	71.71	0.00	0.0%	0.04
Avg	*45.80*	*45.80*	*0.0%*	*0.90*	*45.80*	*45.79*	*45.80*	*0.00*	*0.0%*	*1.46*	*45.80*	*45.80*	*45.80*	*0.00*	*0.0%*	*0.05*
R6/144/2	**35.67**	35.67	0.0%	0.18	**35.67**	35.67	35.67	0.00	0.0%	1.76	**35.67**	35.67	35.67	0.00	0.0%	0.10
R6/288/2	**52.98**	52.98	0.0%	1.61	**52.98**	52.98	52.98	0.00	0.0%	4.72	**52.98**	52.98	52.98	0.00	0.0%	0.08
R6/432/2	**60.98**	60.98	0.0%	0.44	**60.98**	60.98	60.98	0.00	0.0%	5.08	**60.98**	60.98	60.98	0.00	0.0%	0.09
R6/576/2	**63.70**	63.70	0.0%	1.83	**63.70**	63.70	63.70	0.00	0.0%	3.90	**63.70**	63.70	63.70	0.00	0.0%	0.14
R6/144/3	**45.58**	45.58	0.0%	0.38	**45.58**	45.58	45.58	0.00	0.0%	1.82	**45.58**	45.58	45.58	0.00	0.0%	0.08
R6/288/3	**63.83**	63.83	0.0%	1.22	**63.83**	63.83	63.83	0.00	0.0%	2.02	**63.83**	63.83	63.83	0.00	0.0%	0.08
R6/432/3	**68.66**	68.66	0.0%	3.44	**68.66**	68.66	68.66	0.00	0.0%	1.90	**68.66**	68.66	68.66	0.00	0.0%	0.06
R6/576/3	**69.85**	69.85	0.0%	4.78	**69.85**	69.85	69.85	0.00	0.0%	2.16	**69.85**	69.85	69.85	0.00	0.0%	0.07
R6/144/4	**52.21**	52.21	0.0%	0.48	**52.21**	52.16	52.20	0.02	0.0%	0.78	**52.21**	52.21	52.21	0.00	0.0%	0.04
R6/288/4	**69.09**	69.09	0.0%	0.74	**69.09**	69.03	69.08	0.02	0.0%	1.04	**69.09**	69.09	69.09	0.00	0.0%	0.04
R6/432/4	**72.29**	72.29	0.0%	1.43	**72.29**	71.68	72.13	0.24	0.2%	1.10	**72.29**	72.10	72.15	0.08	0.2%	0.05
R6/576/4	**72.85**	72.85	0.0%	2.66	**72.85**	72.18	72.54	0.22	0.4%	1.24	**72.85**	72.53	72.77	0.14	0.1%	0.06
R6/144/5	**58.05**	58.05	0.0%	0.52	**58.05**	58.05	58.05	0.00	0.0%	0.56	**58.05**	58.05	58.05	0.00	0.0%	0.04
R6/288/5	**72.50**	72.50	0.0%	0.43	**72.50**	72.50	72.50	0.00	0.0%	0.62	**72.50**	72.50	72.50	0.00	0.0%	0.05
R6/432/5	**74.72**	74.72	0.0%	0.64	**74.72**	74.30	74.55	0.21	0.2%	0.62	**74.72**	74.72	74.72	0.00	0.0%	0.04
R6/576/5	**75.08**	75.08	0.0%	0.72	**75.08**	75.08	75.08	0.00	0.0%	0.73	**75.08**	75.08	75.08	0.00	0.0%	0.07
Avg	*63.00*	*63.00*	*0.0%*	*1.34*	*63.00*	*62.89*	*62.96*	*0.04*	*0.1%*	*1.88*	*63.00*	*62.97*	*62.99*	*0.01*	*0.0%*	*0.07*

Table 5: Results of the Heuristic Methods on the Instances with 9 Customers

Instance	Enhanced BBC				ILS-IpOpt						ILS-MCS					
	z_{lb}	z_{ub}	Gap	Found Time(s)	Best Obj	Worst Obj	Avg. Obj	Std. Dev.	Gap	Avg. Time(s)	Best Obj	Worst Obj	Avg. Obj	Std. Dev.	Gap	Avg. Time(s)
C9/116/2	**36.32**	36.32	0.0%	2.85	**36.32**	36.32	36.32	0.00	0.0%	3.94	**36.32**	36.32	36.32	0.00	0.0%	0.05
C9/232/2	**63.68**	63.69	0.0%	5.28	**63.68**	63.64	63.67	0.02	0.0%	23.33	**63.68**	63.13	63.54	0.24	0.2%	0.25
C9/348/2	**80.57**	85.60	6.2%	698.65	**80.57**	80.57	80.57	0.00	6.2%	84.87	**80.57**	80.57	80.57	0.00	6.2%	0.54
C9/464/2	**90.53**	109.26	20.7%	148.67	**90.53**	90.53	90.53	0.00	20.7%	57.13	**90.53**	90.53	90.53	0.00	20.7%	0.44
C9/116/3	**46.47**	46.47	0.0%	6.11	**46.47**	46.47	46.47	0.00	0.0%	6.96	**46.47**	46.47	46.47	0.00	0.0%	0.14
C9/232/3	**81.46**	81.47	0.0%	522.02	**81.46**	81.46	81.46	0.00	0.0%	9.92	**81.46**	81.46	81.46	0.00	0.0%	0.30
C9/348/3	**98.78**	101.73	3.0%	6381.63	**98.78**	98.76	98.78	0.01	3.0%	15.01	**98.78**	98.78	98.78	0.00	3.0%	0.34
C9/464/3	**106.82**	127.48	19.3%	598.13	**106.86**	106.86	106.86	0.00	19.3%	20.45	**106.86**	106.86	106.86	0.00	19.3%	0.52
C9/116/4	**56.44**	56.44	0.0%	11.98	**56.44**	55.58	56.12	0.29	0.6%	5.21	**56.44**	56.06	56.02	0.12	0.7%	0.32
C9/232/4	**95.26**	95.27	0.0%	519.61	**95.26**	95.26	95.26	0.00	0.0%	8.37	**95.26**	95.26	95.26	0.00	0.0%	0.26
C9/348/4	**110.96**	110.97	0.0%	1474.63	**110.96**	110.96	110.96	0.00	0.0%	9.60	**110.96**	110.96	110.96	0.00	0.0%	0.28
C9/464/4	**116.98**	116.99	0.0%	2345.14	**116.98**	116.98	116.98	0.00	0.0%	11.31	**116.98**	116.98	116.98	0.00	0.0%	0.49
C9/116/5	**66.17**	66.18	0.0%	14.80	**66.17**	65.75	66.09	0.17	0.1%	3.62	**66.17**	66.17	66.17	0.00	0.0%	0.12
C9/232/5	**105.21**	105.22	0.0%	77.77	**105.20**	105.12	105.17	0.04	0.0%	3.83	**105.20**	105.20	105.20	0.00	0.0%	0.15
C9/348/5	**118.94**	118.95	0.0%	216.98	**118.94**	118.71	118.89	0.09	0.1%	4.20	**118.94**	118.94	118.94	0.00	0.0%	0.18
C9/464/5	**123.00**	123.01	0.0%	1185.06	**123.00**	122.51	122.82	0.18	0.2%	4.63	**123.00**	123.00	123.00	0.00	0.0%	0.21
Avg	*87.35*	*90.31*	*3.1%*	*888.08*	*87.35*	*87.22*	*87.31*	*0.05*	*3.1%*	*17.03*	*87.35*	*87.29*	*87.32*	*0.02*	*3.1%*	*0.29*
R9/176/2	**40.39**	40.39	0.0%	3.58	**40.39**	40.39	40.39	0.00	0.0%	4.85	**40.39**	40.39	40.39	0.00	0.0%	0.04
R9/352/2	**64.92**	64.92	0.0%	164.35	**64.92**	64.92	64.92	0.00	0.0%	14.65	**64.92**	64.92	64.92	0.00	0.0%	0.18
R9/528/2	**73.99**	87.73	18.6%	101.08	**73.99**	73.49	73.89	0.20	18.7%	21.10	**73.99**	73.99	73.99	0.00	18.6%	0.40
R9/704/2	76.71	93.19	21.5%	1165.03	**76.93**	76.67	76.88	0.10	21.2%	31.38	**76.93**	76.71	76.88	0.10	21.2%	0.51
R9/176/3	**56.12**	56.12	0.0%	4.75	**56.12**	55.95	56.05	0.08	0.1%	5.80	**56.12**	56.12	56.12	0.00	0.0%	0.13
R9/352/3	**80.09**	80.10	0.0%	1049.94	**80.09**	80.09	80.09	0.00	0.0%	9.88	**80.09**	80.09	80.09	0.00	0.0%	0.19
R9/528/3	**86.45**	93.63	8.3%	909.40	**86.45**	86.25	86.41	0.08	8.4%	12.59	**86.45**	86.45	86.45	0.00	8.3%	0.17
R9/704/3	**87.47**	98.33	12.4%	2578.38	**87.47**	87.47	87.47	0.00	12.4%	13.80	**87.47**	87.47	87.47	0.00	12.4%	0.39
R9/176/4	**65.99**	65.99	0.0%	84.84	**65.99**	65.99	65.99	0.00	0.0%	5.54	**65.99**	65.99	65.99	0.00	0.0%	0.08
R9/352/4	**88.46**	88.47	0.0%	203.20	**88.46**	88.46	88.46	0.00	0.0%	6.25	**88.46**	88.46	88.46	0.00	0.0%	0.14
R9/528/4	**92.57**	92.58	0.0%	454.31	**92.57**	92.57	92.57	0.00	0.0%	8.44	**92.57**	92.57	92.57	0.00	0.0%	0.17
R9/704/4	**93.20**	93.21	0.0%	1351.38	**93.20**	93.20	93.20	0.00	0.0%	9.96	**93.20**	93.10	93.18	0.04	0.0%	0.37
R9/176/5	**74.14**	74.15	0.0%	50.44	**74.14**	73.79	73.95	0.16	0.3%	3.84	**74.14**	74.14	74.14	0.00	0.0%	0.15
R9/352/5	**93.92**	93.93	0.0%	96.85	**93.92**	92.76	93.58	0.46	0.4%	4.79	**93.92**	93.92	93.92	0.00	0.0%	0.18
R9/528/5	**96.29**	96.30	0.0%	813.91	96.14	95.96	96.04	0.08	0.3%	4.91	**96.29**	96.05	96.23	0.10	0.1%	0.22
R9/704/5	**96.55**	96.56	0.0%	729.64	95.92	95.42	95.73	0.19	0.9%	5.45	**96.55**	96.21	96.47	0.15	0.1%	0.20
Avg	*79.20*	*82.23*	*3.8%*	*610.07*	*79.17*	*78.96*	*79.10*	*0.09*	*3.9%*	*10.20*	*79.22*	*79.16*	*79.20*	*0.02*	*3.8%*	*0.22*

Table 6: Results of the Heuristic Methods on the Instances with 25 Customers

Instance	Enhanced BBC				ILS-IpOpt						ILS-MCS					
	z_{lb}	z_{ub}	Gap	Found Time(s)	Best Obj	Worst Obj	Avg. Obj	Std. Dev.	Gap	Avg. Time(s)	Best Obj	Worst Obj	Avg. Obj	Std. Dev.	Gap	Avg. Time(s)
C25/130/2	59.34	74.02	24.7%	3564.46	**65.76**	65.76	65.76	0.00	12.6%	15.75	**65.76**	65.68	65.71	0.03	12.7%	0.43
C25/260/2	122.40	145.37	18.8%	5446.90	**129.02**	129.02	129.02	0.00	12.7%	86.95	**129.02**	129.02	129.02	0.00	12.7%	1.66
C25/390/2	168.38	224.03	33.0%	4652.80	**173.93**	173.20	173.78	0.29	28.9%	411.95	173.89	173.56	173.74	0.15	28.9%	2.12
C25/520/2	199.68	236.35	18.4%	453.42	**205.61**	205.47	205.55	0.07	15.0%	474.28	205.45	205.45	205.45	0.00	15.0%	8.60
C25/130/3	85.33	109.74	28.6%	3211.66	**90.46**	90.46	90.46	0.00	21.3%	37.31	**90.46**	90.43	90.39	0.03	21.4%	0.85
C25/260/3	167.26	195.53	16.9%	5301.15	**171.12**	171.12	171.12	0.00	14.3%	266.46	171.10	170.98	171.01	0.05	14.3%	2.20
C25/390/3	215.61	252.67	17.2%	6877.86	**221.67**	221.43	221.57	0.12	14.0%	732.88	221.60	221.42	221.50	0.08	14.1%	3.66
C25/520/3	250.22	296.36	18.4%	2.29	**253.94**	253.45	253.80	0.18	16.8%	785.75	253.74	253.70	253.73	0.02	16.8%	41.64
C25/130/4	102.68	139.19	35.6%	5740.65	**110.72**	110.72	110.72	0.00	25.7%	36.86	**110.72**	110.64	110.69	0.03	25.8%	0.62
C25/260/4	192.78	234.26	21.5%	1883.23	**206.37**	206.32	206.35	0.02	13.5%	222.62	**206.37**	206.29	206.25	0.06	13.6%	3.35
C25/390/4	246.15	366.80	49.0%	807.27	**258.83**	258.58	258.74	0.08	41.8%	541.21	258.74	258.59	258.68	0.05	41.8%	9.94
C25/520/4	270.04	429.75	59.1%	3379.70	290.21	289.01	289.52	0.45	48.4%	495.86	**290.24**	289.85	290.13	0.16	48.1%	20.36
C25/130/5	119.61	162.42	35.8%	6073.63	**129.39**	129.39	129.39	0.00	25.5%	50.34	**129.39**	129.31	129.36	0.03	25.6%	1.03
C25/260/5	224.62	266.59	18.7%	5440.23	**233.75**	233.58	233.67	0.07	14.1%	293.30	**233.75**	233.73	233.74	0.01	14.1%	4.06
C25/390/5	271.47	422.02	55.5%	292.13	287.93	287.62	287.76	0.11	46.7%	555.85	**287.98**	287.93	287.97	0.02	46.5%	12.05
C25/520/5	303.75	451.31	48.6%	6410.86	318.16	317.42	317.88	0.26	42.0%	454.29	**318.35**	318.18	318.29	0.07	41.8%	23.64
Avg	*187.46*	*250.40*	*31.2%*	*3721.14*	*196.68*	*196.41*	*196.57*	*0.10*	*24.6%*	*341.35*	*196.66*	*196.55*	*196.60*	*0.05*	*24.6%*	*8.51*
R25/240/2	**77.12**	79.59	3.2%	292.13	**77.12**	77.12	77.12	0.00	3.2%	22.28	**77.12**	77.12	77.12	0.00	3.2%	0.50
R25/480/2	124.86	150.88	20.8%	6410.86	**128.05**	125.76	127.59	0.92	18.3%	160.66	**128.05**	125.76	126.33	0.99	19.4%	1.95
R25/720/2	145.95	199.02	36.4%	6261.31	**155.73**	153.97	154.92	0.68	28.5%	308.92	**155.73**	155.65	155.71	0.03	27.8%	11.68
R25/960/2	158.29	234.43	48.1%	2973.17	**169.58**	169.28	169.47	0.11	38.3%	704.35	**169.58**	169.28	169.48	0.12	38.3%	52.42
R25/240/3	103.68	115.36	11.3%	5068.15	**104.50**	104.50	104.50	0.00	10.4%	51.58	**104.50**	104.49	104.50	0.00	10.4%	0.98
R25/480/3	164.02	197.97	20.7%	2922.62	**168.56**	167.79	168.36	0.29	17.6%	311.35	**168.56**	167.65	168.12	0.45	17.8%	6.28
R25/720/3	170.60	298.91	75.2%	5438.71	**195.07**	194.66	194.88	0.16	53.4%	771.30	**195.07**	195.07	195.07	0.00	53.2%	40.34
R25/960/3	172.43	323.48	87.6%	4997.29	**205.13**	204.33	204.82	0.37	57.9%	2096.38	**205.13**	204.68	204.96	0.19	57.8%	38.25
R25/240/4	126.02	150.89	19.7%	2996.42	129.95	128.30	129.37	0.66	16.6%	112.36	**130.04**	129.95	129.97	0.04	16.1%	1.53
R25/480/4	175.47	297.17	69.4%	4339.86	**197.36**	196.46	197.05	0.39	50.8%	623.25	**197.36**	195.98	197.02	0.60	50.8%	17.18
R25/720/4	201.27	323.41	60.7%	5128.41	**222.08**	221.33	221.80	0.35	45.8%	600.91	**222.08**	173.02	209.67	21.16	54.2%	8.70
R25/960/4	193.57	323.48	67.1%	935.92	**228.49**	227.71	228.11	0.34	41.8%	745.46	**228.49**	228.04	228.38	0.19	41.6%	28.22
R25/240/5	143.08	214.78	50.1%	3918.87	**152.72**	152.36	152.58	0.18	40.8%	106.43	**152.72**	152.36	152.54	0.18	40.8%	2.37
R25/480/5	197.44	319.40	61.8%	6487.38	220.45	219.56	220.14	0.31	45.1%	519.73	**220.85**	220.61	220.79	0.10	44.7%	6.89
R25/720/5	219.05	323.41	47.6%	5706.35	240.10	239.89	240.01	0.08	34.7%	611.18	**240.15**	240.03	240.06	0.05	34.7%	13.86
R25/960/5	224.28	323.48	44.2%	1834.97	**243.91**	243.15	243.59	0.25	32.8%	446.86	**243.91**	243.83	243.88	0.03	32.6%	26.37
Avg	*162.32*	*242.23*	*45.2%*	*4107.03*	*177.43*	*176.64*	*177.14*	*0.32*	*33.5%*	*512.06*	*177.46*	*173.97*	*176.47*	*1.51*	*34.0%*	*16.09*

Table 7: Results of the Heuristic Methods on the Instances with 50 Customers

Instance	Enhanced BBC				ILS-IpOpt						ILS-MCS					
	z_{lb}	z_{ub}	Gap	Found Time(s)	Best Obj	Worst Obj	Avg. Obj	Std. Dev.	Gap	Avg. Time(s)	Best Obj	Worst Obj	Avg. Obj	Std. Dev.	Gap	Avg. Time(s)
C50/200/2	**108.15**	111.83	3.4%	4187.55	**108.15**	108.15	108.15	0.00	3.4%	17.23	**108.15**	107.72	108.06	0.17	3.5%	1.24
C50/400/2	201.60	217.38	7.8%	9569.79	201.21	201.21	201.21	0.00	8.0%	78.11	**201.62**	195.56	200.05	2.26	8.7%	6.70
C50/600/2	264.52	300.17	13.5%	2466.26	**265.07**	264.64	264.98	0.19	13.3%	279.39	264.86	264.39	264.63	0.15	13.4%	24.38
C50/800/2	299.99	368.25	22.8%	13196.10	**307.82**	307.78	307.80	0.02	19.6%	694.30	307.74	307.18	307.59	0.21	19.7%	54.91
C50/200/3	**150.04**	157.44	4.9%	7542.08	**150.04**	150.04	150.04	0.00	4.9%	63.89	**150.04**	149.68	149.94	0.14	5.0%	2.37
C50/400/3	262.42	299.58	14.2%	8407.15	**264.77**	264.37	264.58	0.14	13.2%	376.36	264.62	264.36	264.56	0.10	13.2%	12.64
C50/600/3	327.75	396.57	21.0%	8329.35	**338.25**	337.95	338.09	0.11	17.3%	1117.80	337.96	337.44	337.77	0.18	17.4%	42.27
C50/800/3	369.76	474.83	28.4%	9892.01	**383.02**	382.41	382.82	0.26	24.0%	1447.34	382.95	380.88	382.15	0.71	24.3%	65.38
C50/200/4	185.49	205.75	10.9%	1964.50	**188.00**	183.88	187.18	1.84	9.9%	82.55	**188.00**	187.83	187.97	0.07	9.5%	4.03
C50/400/4	309.01	366.19	18.5%	7351.85	**320.96**	319.33	320.47	0.65	14.3%	658.74	320.89	320.36	320.57	0.20	14.2%	25.28
C50/600/4	358.73	636.99	77.6%	7053.37	**397.76**	397.49	397.69	0.11	60.2%	1736.18	397.70	397.32	397.46	0.13	60.3%	62.52
C50/800/4	413.78	557.14	34.6%	14274.40	**442.20**	440.98	441.75	0.52	26.1%	1856.92	441.89	441.50	441.65	0.14	26.1%	93.31
C50/200/5	212.54	255.65	20.3%	7994.51	**220.75**	220.44	220.69	0.14	15.8%	79.35	**220.75**	220.75	220.75	0.00	15.8%	5.38
C50/400/5	354.65	424.88	19.8%	3704.06	**366.63**	365.79	366.16	0.34	16.0%	769.15	366.39	365.66	366.02	0.24	16.1%	22.50
C50/600/5	414.92	732.06	76.4%	12572.80	**447.60**	445.70	446.90	0.74	63.8%	1733.59	447.37	445.20	446.42	0.75	64.0%	127.06
C50/800/5	451.90	843.70	86.7%	10203.50	**490.38**	489.78	490.04	0.25	72.2%	1597.80	490.29	489.57	489.84	0.29	72.2%	223.04
Avg	*292.83*	*396.77*	*28.8%*	*8044.33*	*305.79*	*305.00*	*305.53*	*0.33*	*23.9%*	*786.79*	*305.70*	*304.71*	*305.34*	*0.36*	*24.0%*	*48.31*
R50/258/2	**107.30**	114.65	6.8%	4111.39	**107.30**	107.30	107.30	0.00	6.8%	23.44	**107.30**	106.46	106.99	0.30	7.2%	1.96
R50/516/2	186.73	217.88	16.7%	37.35	**189.38**	189.38	189.38	0.00	15.0%	129.81	**189.38**	189.38	189.38	0.00	15.0%	7.13
R50/774/2	234.27	298.04	27.2%	13362.50	**239.13**	237.27	238.19	0.89	25.1%	392.04	**239.13**	238.13	238.58	0.38	24.9%	39.87
R50/1032/2	259.65	365.12	40.6%	711.63	**269.51**	268.65	269.04	0.31	35.7%	429.95	268.84	268.84	268.84	0.00	35.8%	86.02
R50/258/3	149.03	164.14	10.1%	9418.80	**150.20**	149.93	150.04	0.15	9.4%	58.25	**150.20**	149.93	150.13	0.10	9.3%	4.94
R50/516/3	250.75	299.11	19.3%	4276.60	253.30	251.92	252.75	0.76	18.3%	205.83	**255.12**	251.92	252.85	1.18	18.3%	20.40
R50/774/3	300.89	393.86	30.9%	4175.46	312.14	310.41	311.16	0.80	26.6%	632.69	**312.30**	310.65	311.71	0.59	26.4%	88.21
R50/1032/3	317.11	465.09	46.7%	1305.99	340.37	339.13	339.96	0.52	36.8%	849.22	**340.75**	339.51	340.01	0.44	36.8%	129.44
R50/258/4	185.36	217.73	17.5%	13597.80	**189.59**	189.05	189.46	0.23	14.9%	128.80	189.50	186.54	188.61	1.09	15.4%	6.52
R50/516/4	298.08	373.93	25.4%	9546.10	**312.14**	311.80	312.02	0.13	19.8%	550.89	312.05	311.76	311.99	0.12	19.9%	28.80
R50/774/4	351.39	466.07	32.6%	1173.43	**368.46**	367.00	367.98	0.66	26.7%	1241.61	368.38	367.16	367.92	0.52	26.7%	112.11
R50/1032/4	88.47	703.65	695.3%	8771.05	392.61	391.12	392.02	0.56	79.5%	1679.10	**392.77**	390.81	392.06	0.69	79.5%	158.36
R50/258/5	213.30	267.72	25.5%	11309.10	**224.75**	223.27	224.12	0.58	19.5%	129.06	224.14	220.05	223.12	1.55	20.0%	9.56
R50/516/5	337.13	428.20	27.0%	2848.34	354.17	353.29	353.73	0.41	21.1%	773.95	**354.49**	353.61	353.82	0.34	21.0%	36.85
R50/774/5	80.63	703.56	772.6%	760.71	**410.16**	408.39	409.44	0.69	71.8%	883.70	410.01	408.70	409.23	0.47	71.9%	82.72
R50/1032/5	-	703.65	-	-	430.77	429.38	429.99	0.53	63.6%	1214.92	**431.01**	429.76	430.51	0.47	63.4%	104.34
Avg	*224.01*	*386.40*	*119.6%*	*5693.75*	*284.00*	*282.96*	*283.54*	*0.45*	*30.7%*	*582.70*	*284.09*	*282.70*	*283.48*	*0.51*	*30.7%*	*57.33*

- indicates the enhanced BBC did not find the lower bound within 4 hours.

Table 8: Results of the Heuristic Methods on the Instances with 75 Customers

Instance	Enhanced BBC				ILS-IpOpt						ILS-MCS					
	z_{lb}	z_{ub}	Gap	Found Time(s)	Best Obj	Worst Obj	Avg. Obj	Std. Dev.	Gap	Avg. Time(s)	Best Obj	Worst Obj	Avg. Obj	Std. Dev.	Gap	Avg. Time(s)
C75/350/2	219.71	237.25	8.0%	1130.15	**219.84**	219.84	219.84	0.00	7.9%	63.39	219.83	217.77	219.01	1.01	8.3%	3.77
C75/700/2	352.79	415.66	17.8%	1025.30	**356.71**	355.36	356.03	0.51	16.7%	577.93	356.69	354.88	356.25	0.69	16.7%	39.12
C75/1050/2	401.68	549.48	36.8%	848.15	432.97	430.57	432.18	0.93	27.1%	1033.81	**434.98**	433.03	434.24	0.65	26.5%	169.28
C75/1400/2	343.45	658.70	91.8%	2760.69	**480.11**	479.91	479.95	0.09	37.2%	1759.82	479.71	479.24	479.56	0.17	37.4%	749.61
C75/350/3	276.70	331.11	19.7%	7447.10	**302.49**	299.40	301.47	1.26	9.8%	321.66	**302.49**	299.20	301.55	1.23	9.8%	9.43
C75/700/3	452.47	547.97	21.1%	7284.94	**470.58**	470.47	470.56	0.05	16.5%	712.21	470.30	468.00	469.83	0.91	16.6%	89.84
C75/1050/3	487.73	706.70	44.9%	8692.55	**557.05**	556.83	556.92	0.10	26.9%	1882.33	557.00	551.78	555.83	2.03	27.1%	311.56
C75/1400/3	504.88	830.62	64.5%	4752.86	**593.49**	593.39	593.46	0.05	40.0%	4292.95	593.38	593.04	593.25	0.16	40.0%	781.08
C75/350/4	346.51	409.69	18.2%	11252.50	**369.09**	368.60	368.80	0.22	11.1%	452.02	368.72	366.57	368.07	0.79	11.3%	14.39
C75/700/4	521.91	661.17	26.7%	2305.76	**556.06**	554.91	555.36	0.59	19.1%	1342.58	555.79	554.82	555.36	0.42	19.1%	147.49
C75/1050/4	534.67	830.89	55.4%	12336.20	**640.80**	639.58	640.29	0.48	29.8%	5376.81	640.76	640.07	640.40	0.23	29.7%	459.02
C75/1400/4	-	1292.98	-	-	667.75	666.36	667.02	0.62	93.8%	6539.61	**668.10**	665.28	666.79	1.08	93.9%	1703.65
C75/350/5	379.85	479.11	26.1%	4295.22	426.12	425.95	426.03	0.08	12.5%	397.01	**427.50**	426.99	427.19	0.19	12.2%	30.02
C75/700/5	429.25	748.42	74.4%	12292.90	**628.01**	625.27	626.53	1.22	19.5%	3465.05	627.20	626.26	626.79	0.32	19.4%	157.53
C75/1050/5	-	1268.15	-	-	**706.32**	703.96	705.21	1.00	79.8%	5095.48	705.73	704.03	704.87	0.54	79.9%	623.59
C75/1400/5	-	1332.92	-	-	**727.97**	727.25	727.62	0.35	83.2%	6388.89	727.42	725.05	726.27	0.95	83.5%	2528.49
Avg	*403.97*	*706.30*	*38.9%*	*5878.79*	*508.46*	*507.35*	*507.95*	*0.47*	*33.2%*	*2481.35*	*508.48*	*506.63*	*507.83*	*0.71*	*33.2%*	*488.62*
R75/278/2	124.35	133.24	7.2%	6725.56	**124.65**	124.29	124.59	0.15	6.9%	162.32	124.45	124.18	124.30	0.09	7.2%	4.35
R75/556/2	222.60	258.47	16.1%	1164.84	**223.60**	223.60	223.60	0.00	15.6%	227.82	223.43	222.43	222.91	0.35	16.0%	24.87
R75/834/2	276.23	359.00	30.0%	1642.55	283.37	283.10	283.30	0.11	26.7%	869.47	**284.34**	282.53	283.15	0.73	26.8%	110.56
R75/1112/2	310.53	446.71	43.9%	764.92	**325.33**	323.77	324.12	0.60	37.8%	1045.26	325.09	322.10	324.18	1.20	37.8%	372.77
R75/278/3	175.90	201.20	14.4%	3468.55	176.94	176.94	176.94	0.00	13.7%	111.51	**177.83**	175.87	177.01	0.65	13.7%	8.69
R75/556/3	294.48	359.75	22.2%	6331.78	**304.33**	301.37	303.48	1.14	18.5%	658.49	303.26	301.29	302.39	0.78	19.0%	64.19
R75/834/3	362.49	489.52	35.0%	2756.88	**382.07**	379.21	380.92	1.26	28.5%	1568.49	381.66	379.13	380.34	0.87	28.7%	288.20
R75/1112/3	341.76	585.26	71.2%	4025.99	**424.71**	420.94	422.29	1.44	38.6%	2446.83	424.51	423.49	424.13	0.46	38.0%	392.47
R75/278/4	221.67	259.96	17.3%	7526.55	**226.89**	226.08	226.57	0.45	14.7%	216.26	**226.89**	226.89	226.89	0.00	14.6%	14.66
R75/556/4	358.25	447.45	24.9%	5503.34	**377.74**	376.12	376.96	0.73	18.7%	1390.27	377.69	377.19	377.54	0.19	18.5%	134.58
R75/834/4	419.95	584.93	39.3%	10776.90	459.25	457.82	458.61	0.56	27.5%	2039.94	**459.28**	458.10	458.56	0.45	27.6%	300.55
R75/1112/4	435.22	689.20	58.4%	9877.16	**498.07**	496.98	497.55	0.40	38.5%	4161.92	497.52	495.51	496.75	0.75	38.7%	513.72
R75/278/5	212.45	314.96	48.2%	2309.74	**266.96**	265.66	266.23	0.56	18.3%	388.32	266.78	264.08	265.82	0.96	18.5%	27.79
R75/556/5	347.97	528.47	51.9%	13930.60	438.42	433.65	437.37	2.08	20.8%	1548.15	**438.51**	435.74	437.80	1.03	20.7%	83.63
R75/834/5	-	937.55	-	-	516.76	516.24	516.56	0.22	81.5%	2647.52	**517.31**	516.69	516.95	0.21	81.4%	269.43
R75/1112/5	183.97	1051.83	471.7%	11955.10	**555.15**	552.55	553.96	1.10	89.9%	3790.63	554.65	553.40	553.88	0.45	89.9%	617.68
Avg	*285.85*	*477.97*	*63.4%*	*5917.36*	*349.02*	*347.40*	*348.32*	*0.67*	*31.0%*	*1454.58*	*348.95*	*347.41*	*348.29*	*0.57*	*31.1%*	*201.76*

- indicates the enhanced BBC did not find the lower bound within 4 hours.

Table 9: Results of the Heuristic Methods on the Instances with 100 Customers

Instance	Enhanced BBC				ILS-IpOpt						ILS-MCS					
	z_{lb}	z_{ub}	Gap	Found Time(s)	Best Obj	Worst Obj	Avg. Obj	Std. Dev.	Gap	Avg. Time(s)	Best Obj	Worst Obj	Avg. Obj	Std. Dev.	Gap	Avg. Time(s)
C100/350/2	201.92	242.39	20.0%	7086.88	**219.83**	214.19	218.10	2.54	11.1%	172.26	**219.83**	219.83	219.83	0.00	10.3%	7.41
C100/700/2	**373.94**	430.98	15.3%	2856.86	371.59	370.73	371.14	0.30	16.1%	459.56	371.88	362.47	369.52	3.56	16.6%	96.50
C100/1050/2	463.14	584.84	26.3%	4829.98	464.49	460.25	463.33	1.75	26.2%	1254.73	**466.37**	462.30	465.33	1.53	25.7%	387.65
C100/1400/2	449.44	718.88	60.0%	4292.64	531.57	524.71	528.05	2.80	36.1%	2960.65	**531.65**	523.44	526.61	3.05	36.5%	1584.70
C100/350/3	245.81	338.19	37.6%	6578.18	**305.40**	305.09	305.15	0.14	10.8%	150.23	304.83	304.56	304.65	0.10	11.0%	15.82
C100/700/3	472.00	584.28	23.8%	8219.06	**502.68**	502.66	502.67	0.01	16.2%	973.22	502.66	502.14	502.37	0.17	16.3%	94.14
C100/1050/3	556.83	776.93	39.5%	11234.00	**618.20**	611.87	616.79	2.77	26.0%	1853.06	613.20	610.86	612.29	0.78	26.9%	511.40
C100/1400/3	485.26	930.45	91.7%	12887.00	**679.53**	679.23	679.35	0.16	37.0%	3337.05	679.10	673.35	677.33	2.31	37.4%	1548.92
C100/350/4	-	432.26	-	-	**384.20**	382.16	383.09	0.93	12.8%	491.34	382.91	382.53	382.69	0.13	13.0%	26.37
C100/700/4	-	722.39	-	-	605.95	605.26	605.60	0.30	19.3%	2030.85	**607.71**	605.11	606.57	0.88	19.1%	279.85
C100/1050/4	-	937.77	-	-	**730.55**	728.57	729.67	0.87	28.5%	5860.85	729.87	728.25	728.98	0.57	28.6%	736.45
C100/1400/4	-	1097.59	-	-	**784.45**	783.26	783.99	0.45	40.0%	7241.30	784.09	781.52	783.21	0.88	40.1%	1238.91
C100/350/5	-	519.14	-	-	**452.92**	449.52	451.59	1.40	15.0%	713.22	452.49	450.04	451.71	0.87	14.9%	56.90
C100/700/5	-	1169.26	-	-	**703.80**	702.65	703.14	0.43	66.3%	4003.98	702.86	702.37	702.53	0.18	66.4%	290.40
C100/1050/5	-	1065.74	-	-	821.94	819.61	821.23	0.93	29.8%	7162.67	**822.04**	819.74	821.15	0.79	29.8%	683.48
C100/1400/5	-	1688.75	-	-	866.02	860.68	864.27	2.15	95.4%	7243.91	**866.58**	863.01	864.97	1.16	95.2%	2211.23
Avg	*406.04*	*764.99*	*39.3%*	*7248.08*	*565.20*	*562.53*	*564.20*	*1.12*	*30.4%*	*2869.30*	*564.88*	*561.97*	*563.73*	*1.06*	*30.5%*	*610.63*
R100/278/2	139.58	160.96	15.3%	4600.18	**144.70**	144.54	144.60	0.08	11.3%	285.69	144.69	144.16	144.49	0.18	11.4%	20.35
R100/556/2	246.66	288.45	16.9%	4390.93	**251.08**	249.26	249.63	0.81	15.6%	360.39	250.85	249.00	249.69	0.82	15.5%	82.55
R100/834/2	317.27	406.34	28.1%	6740.48	**327.62**	326.38	327.37	0.55	24.1%	788.65	326.99	325.77	326.38	0.55	24.5%	270.72
R100/1112/2	360.02	511.76	42.1%	4661.07	375.71	374.45	374.88	0.48	36.5%	1787.21	**379.14**	374.82	378.08	1.64	35.4%	627.70
R100/278/3	151.56	227.94	50.4%	10298.10	**203.71**	203.26	203.56	0.18	12.0%	260.83	203.69	202.62	203.27	0.38	12.1%	14.69
R100/556/3	-	408.08	-	-	**348.15**	346.55	347.71	0.67	17.4%	1100.21	348.02	345.95	347.01	0.78	17.6%	96.37
R100/834/3	423.89	559.87	32.1%	13092.20	**442.11**	436.34	439.57	2.39	27.4%	2928.94	441.82	440.48	441.40	0.51	26.8%	539.41
R100/1112/3	-	683.48	-	-	**495.84**	494.27	495.08	0.75	38.1%	4431.45	495.20	491.01	493.50	1.43	38.5%	2033.04
R100/278/4	-	291.18	-	-	**257.56**	256.36	257.10	0.48	13.3%	441.35	**257.56**	255.76	256.71	0.70	13.4%	22.00
R100/556/4	387.64	510.74	31.8%	14139.50	**432.71**	425.76	430.91	2.92	18.5%	2216.70	431.93	430.76	431.38	0.44	18.4%	245.30
R100/834/4	-	947.58	-	-	**535.51**	534.47	534.94	0.37	77.1%	4823.02	535.02	528.29	533.16	2.51	77.7%	1267.70
R100/1112/4	184.20	1137.69	517.6%	13717.50	**589.41**	587.28	588.09	0.83	93.5%	6500.78	588.96	585.56	587.92	1.30	93.5%	1804.47
R100/278/5	-	348.96	-	-	**309.75**	309.38	309.60	0.20	12.7%	614.53	309.54	308.57	309.23	0.35	12.8%	50.06
R100/556/5	-	606.72	-	-	**505.38**	501.92	504.21	1.34	20.3%	2190.49	504.46	501.65	503.13	1.22	20.6%	247.86
R100/834/5	51.80	1106.98	2037.1%	5276.28	613.03	609.68	611.40	1.19	81.1%	3364.96	**613.41**	610.93	612.08	0.92	80.9%	581.03
R100/1112/5	-	918.58	-	-	**666.90**	659.06	661.85	3.01	38.8%	4382.88	665.78	662.07	663.96	1.44	38.3%	1085.97
Avg	*251.40*	*569.71*	*307.9%*	*8546.25*	*406.20*	*403.69*	*405.03*	*1.02*	*33.6%*	*2279.88*	*406.07*	*403.59*	*405.09*	*0.95*	*33.6%*	*561.83*

- indicates the enhanced BBC did not find the lower bound within 4 hours.

3.4 Modified Objective Function

Theoretically, the enhanced BBC framework can be adapted to solve the problem where objective function is twice continuously differentiable. Furthermore, this framework can tackle a more general objective function as long as the nonlinear model [SSP] can be solved to optimality. Although the inequalities proposed for estimating the upper bound of the objective value should be revised based on the form of the chosen objective function, we can still employ the linear estimation technique to derive an upper bound.

To derive an upper bound of the modified objective function in terms of arrival time, the inequality (29) in the [EnMP] is revised accordingly. Here we use a simple linear estimation $\sum_{i\in\hat{N}}(p_i y_{i,v} - \frac{p_i}{D_i} a_{i,v})$ as the term on the right hand side of inequality (29). Because the profit will never decrease to zero when the arrival time increases in the case of the modified objective function described in Section 6.5, we set a rule that each customer becomes unprofitable when its profit decreases to 10^{-3}. Therefore, the value of D_i is set to $-\ln(10^{-3}/p_i)/d_i$. With the changes indicated above, the enhanced BBC framework can be used to solve the problem with the modified objective function.

The instances, which were used for the original experiments in this case, are modified by changing the value of delay ratio d_i in the small size instances used for the original objective function. More specifically, we use d_i^{Exp} and d_i^{Lin} to denote the delay ratio in the exponentially decreasing profit function and the linearly decreasing profit function, respectively. The value of d_i^{Exp} is computed as $-\ln(10^{-3}/p_i)/(p_i/d_i^{Lin})$. When the profit of a customer decreases to 10^{-3}, we assume that the profit is zero. Thus, changing the delay ratio this way could keep the deadline of each customer unchanged.

Tables 10-12 show the results obtained by different methods for instances with 6, 9 and 25 customers under the modified objective function. We note that the column "Gap" for ILS-MCS is computed by $(z_{ub} - Avg.Obj)/Avg.Obj \times 100\%$. Boldface letters are used to indicate the best found objective value of each instances. To validate the results obtained by the enhanced BBC, SCIP is employed for the instances with 6 customers. The computing time is set to 4 hours for SCIP and the enhanced BBC, the other parameters of the enhanced BBC and the ILS-MCS are the same as in the experiments for the original objective function.

Table 10: Results of the Exact and Heuristic Methods on the Instances with 6 Customers When the Profit Decreases Exponentially with Arrival Time

	SCIP				Enhanced BBC				ILS-MCS					
Instance	z_{lb}	z_{ub}	Gap	Time (s)	z_{lb}	z_{ub}	Gap	Time (s)	Best Obj	Worst Obj	Avg. Obj	Std. Dev	Gap	Avg. Time(s)
C6/70/2	**14.65**	14.65	0.00%	22.26	**14.65**	14.65	0.0%	0.12	**14.65**	14.65	14.65	0.00	0.0%	0.04
C6/140/2	**26.18**	26.18	0.00%	51.37	**26.18**	26.18	0.0%	4.00	**26.18**	26.18	26.18	0.00	0.0%	0.03
C6/210/2	**31.63**	31.63	0.00%	133.64	**31.63**	31.63	0.0%	5.61	**31.63**	31.63	31.63	0.00	0.0%	0.03
C6/280/2	**34.21**	34.21	0.00%	229.33	**34.21**	34.21	0.0%	11.50	**34.21**	34.21	34.21	0.00	0.0%	0.03
C6/70/3	**18.27**	18.27	0.00%	17.94	**18.27**	18.27	0.0%	0.61	**18.27**	18.27	18.27	0.00	0.0%	0.05
C6/140/3	**32.97**	32.97	0.00%	49.38	**32.97**	32.97	0.0%	2.95	**32.97**	32.97	32.97	0.00	0.0%	0.06
C6/210/3	**39.56**	39.56	0.00%	132.42	**39.56**	39.56	0.0%	5.09	**39.56**	39.56	39.56	0.00	0.0%	0.06
C6/280/3	**42.53**	42.53	0.00%	204.06	**42.53**	42.53	0.0%	7.88	**42.53**	42.53	42.53	0.00	0.0%	0.05
C6/70/4	**21.83**	21.83	0.00%	10.56	**21.83**	21.83	0.0%	0.97	**21.83**	21.83	21.83	0.00	0.0%	0.09
C6/140/4	**39.49**	39.49	0.00%	24.62	**39.49**	39.49	0.0%	2.85	**39.49**	39.49	39.49	0.00	0.0%	0.06
C6/210/4	**46.92**	46.92	0.00%	33.99	**46.92**	46.92	0.0%	3.66	**46.92**	46.92	46.92	0.00	0.0%	0.05
C6/280/4	**50.28**	50.28	0.00%	42.97	**50.28**	50.28	0.0%	6.53	**50.28**	50.28	50.28	0.00	0.0%	0.08
C6/70/5	**25.25**	25.25	0.00%	3.63	**25.25**	25.25	0.0%	0.49	**25.25**	25.25	25.25	0.00	0.0%	0.04
C6/140/5	**45.65**	45.65	0.00%	3.7	**45.65**	45.65	0.0%	0.97	**45.65**	45.65	45.65	0.00	0.0%	0.05
C6/210/5	**54.17**	54.17	0.00%	2.98	**54.17**	54.17	0.0%	1.08	**54.17**	54.17	54.17	0.00	0.0%	0.07
C6/280/5	**57.87**	57.87	0.00%	3.11	**57.87**	57.87	0.0%	1.10	**57.87**	57.87	57.87	0.00	0.0%	0.06
Avg	*36.34*	*36.34*	*0.0%*	*60.37*	*36.34*	*36.34*	*0.0%*	*3.46*	*36.34*	*36.34*	*36.34*	*0.00*	*0.0%*	*0.05*
R6/144/2	**31.34**	31.34	0.00%	29.43	**31.34**	31.34	0.0%	1.00	**31.34**	31.34	31.34	0.00	0.0%	0.10
R6/288/2	**39.23**	39.23	0.00%	64.04	**39.23**	39.23	0.0%	11.11	**39.23**	39.23	39.23	0.00	0.0%	0.09
R6/432/2	**40.68**	40.68	0.00%	102.51	**40.68**	40.68	0.0%	31.32	**40.68**	40.68	40.68	0.00	0.0%	0.16
R6/576/2	**41.04**	41.04	0.00%	98.63	**41.04**	41.04	0.0%	52.88	**41.04**	41.04	41.04	0.00	0.0%	0.71
R6/144/3	**39.05**	39.05	0.00%	21.91	**39.05**	39.05	0.0%	2.13	**39.05**	39.05	39.05	0.00	0.0%	0.08
R6/288/3	**48.99**	48.99	0.00%	31.32	**48.99**	48.99	0.0%	12.21	**48.99**	48.94	48.98	0.02	0.0%	0.10
R6/432/3	**50.86**	50.86	0.00%	36.62	**50.86**	50.86	0.0%	29.99	**50.86**	50.86	50.86	0.00	0.0%	0.15
R6/576/3	**51.16**	51.16	0.00%	64.87	**51.16**	51.16	0.0%	35.40	**51.16**	50.74	50.99	0.23	0.0%	0.74
R6/144/4	**45.78**	45.78	0.00%	8.56	**45.78**	45.78	0.0%	2.25	**45.78**	45.78	45.78	0.00	0.0%	0.05
R6/288/4	**56.80**	56.80	0.00%	10.53	**56.80**	56.80	0.0%	4.89	**56.80**	56.80	56.80	0.00	0.0%	0.08
R6/432/4	**58.35**	58.35	0.00%	21.99	**58.35**	58.35	0.0%	8.03	**58.35**	58.35	58.35	0.00	0.0%	0.11
R6/576/4	**58.61**	58.61	0.00%	13.75	**58.61**	58.61	0.0%	14.22	**58.61**	58.61	58.61	0.00	0.0%	0.51
R6/144/5	**49.95**	49.95	0.00%	1.73	**49.95**	49.95	0.0%	0.97	**49.95**	49.95	49.95	0.00	0.0%	0.05
R6/288/5	**61.54**	61.54	0.00%	0.84	**61.54**	61.54	0.0%	1.82	**61.54**	61.54	61.54	0.00	0.0%	0.06
R6/432/5	**63.12**	63.12	0.00%	0.8	**63.12**	63.12	0.0%	1.64	**63.12**	63.12	63.12	0.00	0.0%	0.08
R6/576/5	**63.39**	63.39	0.00%	6.3	**63.39**	63.39	0.0%	1.34	**63.39**	63.39	63.39	0.00	0.0%	0.38
Avg	*49.99*	*49.99*	*0.00*	*32.11*	*49.99*	*49.99*	*0.00*	*13.20*	*49.99*	*49.96*	*49.98*	*0.02*	*0.00*	*0.22*

Table 11: Results of the Exact and Heuristic Methods on the Instances with 9 Customers When the Profit Decreases Exponentially with Arrival Time

Instance	Enhanced BBC					ILS-MCS					
	z_{lb}	z_{ub}	Gap	Time (s)	Found Time(s)	Best Obj	Worst Obj	Avg. Obj	Std. Dev	Gap	Avg Time(s)
C9/116/2	**31.36**	31.36	0.0%	4.12	2.53	**31.36**	31.36	31.36	0.00	0.0%	0.16
C9/232/2	**46.13**	46.13	0.0%	227.70	11.45	**46.13**	46.13	46.13	0.00	0.0%	0.16
C9/348/2	**50.24**	50.24	0.0%	1064.86	892.40	**50.24**	50.24	50.24	0.00	0.0%	0.19
C9/464/2	**51.17**	51.17	0.0%	1069.13	841.15	**51.17**	51.17	51.17	0.00	0.0%	0.22
C9/116/3	**42.26**	42.26	0.0%	11.29	1.32	**42.26**	42.26	42.26	0.00	0.0%	0.14
C9/232/3	**60.77**	60.77	0.0%	269.36	217.67	**60.77**	60.77	60.77	0.00	0.0%	0.18
C9/348/3	**65.72**	65.73	0.0%	870.92	662.31	**65.72**	65.72	65.72	0.00	0.0%	0.23
C9/464/3	**66.85**	66.85	0.0%	1959.92	1959.91	**66.85**	66.85	66.85	0.00	0.0%	0.23
C9/116/4	**49.69**	49.69	0.0%	22.42	18.38	**49.69**	49.69	49.69	0.00	0.0%	0.18
C9/232/4	**72.73**	72.73	0.0%	291.05	371.29	**72.73**	72.73	72.73	0.00	0.0%	0.18
C9/348/4	**76.83**	76.83	0.0%	935.63	742.54	**76.83**	76.83	76.83	0.00	0.0%	0.27
C9/464/4	**77.86**	77.86	0.0%	1819.32	1288.20	**77.86**	77.86	77.86	0.00	0.0%	0.23
C9/116/5	**55.70**	55.70	0.0%	35.73	15.69	**55.70**	55.70	55.70	0.00	0.0%	0.18
C9/232/5	**80.83**	80.83	0.0%	334.43	212.94	**80.83**	80.83	80.83	0.00	0.0%	0.21
C9/348/5	**85.27**	85.28	0.0%	623.97	386.16	**85.27**	85.27	85.27	0.00	0.0%	0.23
C9/464/5	**86.36**	86.36	0.0%	1114.74	131.22	**86.36**	86.36	86.36	0.00	0.0%	0.27
Avg	*62.49*	*62.49*	*0.0%*	665.91	484.70	*62.49*	*62.49*	*62.49*	*0.00*	*0.0%*	*0.20*
R9/176/2	**34.40**	34.40	0.0%	13.42	0.25	**34.40**	34.40	34.40	0.00	0.0%	0.10
R9/352/2	**40.12**	40.13	0.0%	856.99	851.94	**40.12**	40.12	40.12	0.00	0.0%	0.21
R9/528/2	**40.98**	40.99	0.0%	1694.78	457.14	**40.98**	40.98	40.98	0.00	0.0%	0.43
R9/704/2	**41.12**	41.12	0.0%	9818.63	5836.98	**41.12**	41.10	41.12	0.01	0.0%	0.51
R9/176/3	**45.21**	45.21	0.0%	64.26	23.77	**45.21**	45.21	45.21	0.00	0.0%	0.13
R9/352/3	**52.99**	52.99	0.0%	1522.51	788.92	**52.99**	52.99	52.99	0.00	0.0%	0.22
R9/528/3	**53.93**	53.93	0.0%	3575.88	2793.64	**53.93**	53.93	53.93	0.00	0.0%	0.34
R9/704/3	**54.04**	76.65	41.8%	14400.10	8833.69	**54.04**	54.04	54.04	0.00	41.8%	0.40
R9/176/4	**53.23**	53.23	0.0%	101.26	53.05	**53.23**	53.23	53.23	0.00	0.0%	0.16
R9/352/4	**62.01**	62.02	0.0%	1944.43	1219.53	**62.01**	62.01	62.01	0.00	0.0%	0.20
R9/528/4	**62.86**	62.86	0.0%	3102.69	2473.74	**62.86**	62.86	62.86	0.00	0.0%	0.37
R9/704/4	**62.96**	76.41	21.4%	14400.10	12922.50	**62.96**	62.96	62.96	0.00	21.4%	0.51
R9/176/5	**60.67**	60.67	0.0%	105.15	101.79	**60.67**	60.67	60.67	0.00	0.0%	0.17
R9/352/5	**69.08**	69.09	0.0%	1021.88	874.24	**69.08**	69.08	69.08	0.00	0.0%	0.19
R9/528/5	**69.85**	69.85	0.0%	1364.38	1070.18	**69.85**	69.85	69.85	0.00	0.0%	0.27
R9/704/5	**69.95**	69.96	0.0%	6089.51	5914.70	**69.95**	69.95	69.95	0.00	0.0%	0.53
Avg	*54.59*	*56.84*	*4.0%*	3754.75	2763.50	*54.59*	*54.59*	*54.59*	*0.00*	*4.0%*	*0.30*

Table 12: Results of the Exact and Heuristic Methods on the Instances with 25 Customers When the Profit Decreases Exponentially with Arrival Time

	Enhanced BBC				ILS-MCS					
Instance	z_{lb}	z_{ub}	Gap	Found Time(s)	Best Obj	Worst Obj	Avg. Obj	Std. Dev	Gap	Avg. Time(s)
C25/130/2	56.21	66.75	18.7%	52.68	**56.46**	54.65	55.74	0.99	19.8%	0.55
C25/260/2	54.72	135.42	147.5%	3.72	**95.35**	95.35	95.35	0.00	42.0%	0.86
C25/390/2	94.73	189.39	99.9%	6026.02	**113.36**	113.36	113.36	0.00	67.1%	1.19
C25/520/2	48.36	231.01	377.7%	6.22	**119.13**	118.97	119.10	0.07	94.0%	3.08
C25/130/3	**77.76**	92.27	18.7%	9416.93	**77.76**	77.49	77.64	0.14	18.8%	0.69
C25/260/3	116.13	179.96	55.0%	10645.70	**126.19**	126.19	126.19	0.00	42.6%	1.02
C25/390/3	72.36	242.47	235.1%	3.55	**143.42**	143.42	143.42	0.00	69.1%	1.92
C25/520/3	130.14	332.88	155.8%	3159.38	**148.20**	148.05	148.17	0.07	124.7%	4.28
C25/130/4	90.75	114.99	26.7%	12552.00	**94.93**	94.86	94.88	0.03	21.2%	1.00
C25/260/4	118.95	223.79	88.1%	755.06	**150.51**	150.46	150.49	0.03	48.7%	2.54
C25/390/4	137.34	343.53	150.1%	6994.81	**168.08**	167.33	167.90	0.32	104.6%	4.08
C25/520/4	91.42	391.40	328.1%	101.82	**172.37**	171.64	172.22	0.33	127.3%	5.88
C25/130/5	97.26	142.22	46.2%	6952.19	**110.55**	110.44	110.48	0.06	28.7%	1.47
C25/260/5	120.37	328.35	172.8%	3417.73	**171.72**	171.72	171.72	0.00	91.2%	1.90
C25/390/5	157.40	388.42	146.8%	10230.60	**188.93**	188.40	188.70	0.23	105.8%	3.59
C25/520/5	107.23	391.40	265.0%	86.16	**192.84**	192.23	192.62	0.23	103.2%	3.96
Avg	*98.20*	*237.14*	*145.8%*	*4400.29*	*133.11*	*132.79*	*133.00*	*0.16*	*69.3%*	*2.37*
R25/240/2	54.27	80.00	47.4%	1271.20	**55.93**	55.93	55.93	0.00	43.0%	0.44
R25/480/2	43.03	147.39	242.5%	7539.86	**70.47**	70.47	70.47	0.00	109.1%	0.96
R25/720/2	24.48	197.12	705.4%	1089.96	**72.28**	72.28	72.28	0.00	172.7%	5.52
R25/960/2	29.15	265.41	810.4%	12975.80	**72.46**	72.45	72.46	0.01	266.3%	6.06
R25/240/3	72.99	112.12	53.6%	3053.14	**75.48**	75.48	75.48	0.00	48.5%	0.60
R25/480/3	77.41	240.65	210.9%	9037.75	**90.97**	90.97	90.97	0.00	164.5%	2.01
R25/720/3	64.43	265.35	311.8%	2377.60	**92.69**	92.68	92.68	0.01	186.3%	6.70
R25/960/3	22.60	265.41	1074.2%	12740.80	**92.84**	92.82	92.84	0.01	185.9%	11.61
R25/240/4	80.62	144.59	79.4%	9111.92	**93.08**	92.74	93.01	0.15	55.5%	1.09
R25/480/4	80.67	264.48	227.9%	10623.90	**109.35**	109.11	109.29	0.10	142.0%	2.45
R25/720/4	48.86	265.35	443.0%	7089.84	**110.80**	110.67	110.77	0.06	139.5%	9.99
R25/960/4	43.68	265.41	507.6%	8.78	**110.92**	110.82	110.88	0.04	139.4%	8.88
R25/240/5	98.37	175.27	78.2%	8822.56	**109.62**	107.75	108.65	0.90	61.3%	2.88
R25/480/5	59.05	264.48	347.9%	56.23	**126.09**	125.93	126.05	0.07	109.8%	3.50
R25/720/5	42.20	265.35	528.8%	8838.92	**127.43**	126.53	127.06	0.48	108.8%	9.23
R25/960/5	72.49	265.41	266.1%	13077.20	**127.55**	126.63	127.02	0.45	109.0%	10.82
Avg	*57.14*	*217.74*	*370.9%*	*6732.22*	*96.12*	*95.83*	*95.99*	*0.14*	*127.6%*	*5.17*

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