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Productivity-driven physician scheduling in emergency departments

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ABSTRACT The objective of this study is two-fold: to propose an alternative approach for computing the productivity of physicians in emergency departments (EDs); and, to allocate productivity- driven schedules to ED physicians so as to align physician productivity with demand (patient arrivals), without decreasing fairness between physicians, in order to improve patient wait times. Historical data between 2008 and 2017 from the Sacré-Coeur Montreal Hospital ED is analysed and used to predict the demand and to estimate the productivity of each physician. These estimates are incorporated into a mathematical programming model that identifies feasible schedules to physicians that minimise the difference between patientsL demand and physiciansL productivity, along with the violation of physiciansL preferences and fairness in the distribution of shifts. Results on real-world-based data show that when physician productivity is included in the allocation of schedules, demand under-covering is reduced by 10.85% and the fairness between physicians is maintained. However, physiciansL preferences (e.g., sum of the differences between the number of wanted shifts and the number of allocated shifts) deteriorates by 7.61%. By incorporating the productivity of physicians in the scheduling process, we see a reduction in EDs overcrowding and an improvement in the overall quality of health-care services.

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1. Introduction

Physician scheduling plays a critical role in Emergency

Departments (ED) planning. The impact of having the right amount of resources, with the right skills and experience level, can significantly improve the quality of service for patients as well as working conditions for physicians.

In the province of Quebec, specifically, EDs are known to be the most overcrowded in the world (Cooke et al., 2004). This problem leads to undesirable consequences that degrade Quebec's health-care system. Some exam- ples of these consequences are: patients leave emergencies without having seen a physician (Derlet & Richards, 2000), physician productivity is decreased, there is extended dissatisfaction and suffering for the patient, and wait times are excessive. The rate of adults waiting at least 5 h during their last emergency visit in 2016 in Quebec was 44%, twice as high than in the rest of Canada (Quebec, 2017). Reducing this congestion is a high prior- ity to offer better care in EDs in Quebec. The government of Quebec highlighted staff scheduling, among other factors, as a cause this overload (Quebec, 2016). Therefore, to improve this problem, ensuring better plan- ning for ED physicians is one of the major challenges to tackle. (Hoot & Aronsky, 2008). This also true in the United-States (Glass & Anderson, 2018).

Physician planning is a complex task since emer- gency rooms are open 24 h a day, 7 days a week. ED physicians often work night and weekend shifts, while other specialties simply require physicians "on call" during their shifts (Carter & Lapierre, 2001). In addition, ED physicians must diagnose all types of illnesses while other physicians focus on diseases that are specific to their specialty. These features create a variation in their own productivity, which can lead to an impact in ED overcrowding. Although different methods have been proposed to meet the demand in EDs, most of them do not take into account the productivity of physicians. This constitutes a serious limitation, as productivity behaviour is highly vari- able from one physician to another due to different seniority levels or to the work environment.

To better manage ED overcrowding and lack of systematic tools to generate schedules, we propose an innovative approach based on physician productivity and the estimated hourly demand (patient arrivals) embedded in a mathematical model. We analyse the ED patient arrivals to determine the most accurate demand forecasting model. Then, we introduce a physicians' productivity index that seeks to reflect the work behaviour from each physician. This index is based on a *heaviness* classification of each patient (i.e., length of consultation), as well as on an estimate of the average number of patients served at each shift and day of the week. We further propose an optimi- sation model that includes physicians' preferences and availabilities, workload fairness and several work regulations. The objective is to minimise the sum of the differences between patient arrivals and physicians' productivity, along with the violation of physicians' preferences and fairness in the distribu- tion of shifts. Unfairness between physicians is mea- sured as the sum of differences between the number of day shifts and the number of evening and night shifts allocated to the physicians' schedules. By mini- mising this unfairness, the model ensures that the most efficient physicians are not consistently allo- cated to schedules with high demand. Hence, a rotation of shifts between physicians is guaranteed. This project is motivated by a collaboration with Sacré-Cœur Hospital of Montreal (HSCM). We have received the approval of the ethics department of this Hospital to use anonymous patient data from March 2008 to September 2017, which represent 643,000 records in patient files.

The paper is organised as follows. In Section 2, we review related work on patient arrival prediction. We also present a revision on the computation of physi- cian productivity, as well as physician scheduling. In Section 3, we present the methodology to build phy- sician schedules aimed at reducing overcrowding in EDs. Numerical results are presented and discussed in Section 4. Concluding remarks and future work follow in Section 5.

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2. Related work

A general classification of the personnel scheduling process is suggested in Ernst,, Jiang,, Krishnamoorthy, and Sier (2004). This classification contains several modules starting with the demand modelling to determine staffing requirements and ending with the specification of the work to be performed, over a given planning horizon, by each individual in the workforce. In this section, we present a brief review of recent works addressing the different modules used for the design and allocation of productivity-driven physician sche- dules in EDs.

2.1. Patient arrivals

The quality of ED services, often measured by waiting time and length of stay, is significantly affected by an accurate prediction of patient arrivals (Xu, Wong, & Chin, 2013), as decisions involving staff planning and allocation of resources within ED highly depend on these predictions. There has been a significant amount of work done applying different techniques to model and predict the demand in the health-care sector. Time series analysis is among the most popular methods, with appli- cations including the use of exponential smoothing (Boyle et al., 2012; Champion et al., 2007; Jones et al., 2008), autoregressive integrated moving average (ARIMA) models (Boyle et al., 2012; Champion et al.,

2007; Sun., Heng, Seow, & Seow, 2009), univariate and multivariate seasonal autoregressive integrated moving average (SARIMA) models (Jones et al., 2008; Kam, Sung, & Park, 2010), multivariate vector autoregressive (VAR) models (Jones et al., 2009), and generalised autoregressive conditional heteroskedasticity (GARCH) models (jones2002forecasting, 2002). Linear regression models and nonlinear regression models (Boyle et al., 2012; Xu et al., 2013) have also been proposed as alternatives to predict patient visits to EDs. The previous methods have been successfully used in ED forecasting since they allow seasonal modelling by including variables for the day of week, month of year and holidays. They also allow the identification of repeated patterns in the time-series data. Poisson regression models and artificial neural networks are also among the different alternatives used to predict the demand for ED services. Some applications of these techniques are presented in (Jones et al., 2008; McCarthy et al., 2008; Moineddin, Meaney, Agha, Zagorski, & Glazier, 2011; Xu et al., 2013). The reader is referred to (Wargon, Guidet, Hoang, & Hejblum, 2009) for a review on studies designed to pre- dict patient attendance at EDs or walk-in clinics.

2.2. Productivity

In order to meet patient demand in EDs, it is impor- tant to quantify physicians' productivity so as to know the hospital's emergency health capacity. Physicians' productivity is generally defined in the literature by two major indicators: the patients seen per hour (Pt/hr) and the relative value unit (RVU). These indicators study different aspects of physicians' productivity. The ratio Pt/hr denotes the average number of patients seen by a physician per hour without taking into account patients who were handed over at change of shift (Arya, Salovich, Ohman-Strickland, & Merlin, 2010; Leung et al., 2018). On the other hand, RVU measures estimates of physicians' effort and practice expense. They reflect the complexity of tasks, thus technical skills, mental effort and psychological stress (Bhargava & Mishra, 2014). Although these indicators were initially pro- posed as a way of bringing more homogeneity to the health-care reimbursement procedures (Glass & Anderson, 2002), the ratio of Pt/hr and the RVU present some important shortcomings. These are mostly related to changes in medical practice (Storfa & Wilson, 2015). The literature shows various studies to improve physicians' productivity. The physician payment mechanism, for example, by fee-for-service compensation is evaluated in (Innes et al., 2018). According to (Leung et al., 2018) the use of a physician navigator, a team member that assists a physician in activities to reduce the non-clinical workload during a shift, improves the productivity of ED physicians. In (Arya et al., 2010) it is shown that the utilisation of scribes, a person who assists physicians with the clerical aspects of patient care, can help enhance physician productivity. In this paper, we are interested in indicators that capture variability in productivity related to the schedules. None of Pt/hr and RVUs take into consideration the ability of physicians to take new patients based on factors related to the type of shift and the type of day. Only (Dula, Dula, Hamrick, & Wood, 2001) have shown that a long sequence of night shifts might decrease the productivity of physicians. We propose to fill this gap and extend the indicator pt/hr to consider alternative factors that might influence productivity (e.g., type of shift, day of the week).

2.3. Physician scheduling

The body of operations research literature directed to ED care services is extensive (Erhard, Schoenfelder, Fügener, & Brunner, 2017). This literature mainly focuses on strategic decisions related to regional cov- erage and capacity dimensioning for ambulances, and on tactical decisions associated with physician and nurse scheduling (Hulshof, Kortbeek, Boucherie, Hans, & Bakker, 2012). The importance of physician scheduling lies in the implementation of different staffing levels. These are then based on patient arrival rates at different moments within the day and week to decrease patient waiting times and reduce the num- ber of patients that leave the ED without being seen (Green, Soares, Giglio, & Green, 2006). With this in mind (El-Rifai, Garaix, Augusto, & Xie, 2015) intro- duces a stochastic optimisation model for ED physi- cian scheduling. This model takes into account the stochastic nature of patient arrivals to create physi- cian schedules that respond in a robust way to demand variability. Two studies have already addressed a similar physician scheduling problem for the HSCM ED. In (Gendreau et al., 2006), the problem of constructing physician schedules for emergency rooms demonstrates studies from five dif- ferent hospitals (including HSCM). The authors pro- pose a generic form for the constraints encountered in EDs. In (Beaulieu, Ferland, Gendron, & Michelon, 2000), a mathematical programming model was developed to schedule physicians in ED for a planning horizon of 6 months. Although these studies describe the physician scheduling situation in EDs and show that automated approaches can significantly reduce the time and effort required to construct good-quality physician schedules, the pro- posed approaches do not match physicians' produc- tivity with patient demands, which we believe constitutes a powerful way to reduce patients' waiting time.

Some studies propose incorporating the produc- tivity of physicians in the composition of schedules.

An analytic model capable of scheduling providers with different skill profiles and with patients of varying acuity levels is proposed in (Ganguly, Lawrence, & Prather, 2014). In (Savage, Woolford, Weaver, & Wood, 2015), Poisson-based generalised additive models are used to estimate patient arrival rates. A mathematical programming model that incorporates physicians' productivity (computed as the average ratio of the number of patients seen per hour) is proposed to produce an optimal ED shift schedule. Although these studies show that aligning physician productivity with patient arrivals helps to balance staffing costs and to decrease unmet patient demand in EDs, the differences in performance between physicians are not evaluated as the productivity ratio is assumed to be the same for each physician. In addition, these studies do not incorporate work regulations for the composition of schedules, physicians' availabilities, and fairness in the distribution of shifts, creating serious limita- tions that degrade the work–life balance for physi- cians in EDs.

3. Problem definition and formulation

The physician scheduling problem in EDs considers a *planning horizon* including $|D|$ days, where each day $d \in D$ is divided into equal-length *time intervals* $i \in I_d$. We assume that a fixed number of physicians $|P|$ is given and that each physician $p \in P$ is characterised by some preferences and availabilities. Emergencies (E) are typi- cally divided into multiple sections or types. Therefore, we divide the set of *daily shifts* S into the same number of sections. Each shift is also divided into types, depending on the hours of the day they cover (e.g., *day* S_D, *evening* S_E and *night* S_N). In our case, HCSM is divided into two sections: *acute care area* (A) and *fast-track clinic* (F) ($E = \{A, F\}$), and two types of shifts are used (*acute care area* shifts S^A and *fast-track clinic* shifts S^F). Each shift $s \in S$ is characterised by a set of attributes, namely: a start time b_s, a length l_s and the section of the emergency it covers e_s. The demand (given by parameter d_{edi}) denotes the number of patients arriving each day $d \in D$, at each time interval $i \in I_d$, for each emergency section $e \in E$. The objective of the physician scheduling problem in EDs is to allocate feasible schedules to physi- cians while minimising the under-covering and over- covering of demand (number of new patients arriving at each time period), while also taking into account deviations in physician preferences and the fairness in the distribution of shifts.

In this section, we present the methodology to solve the problem under study. First, we describe the methods adopted to forecast the demand and to compute the physicians' productivity. Second, we present the notation and formulation of the optimi- sation model used to generate physician schedules.

3.1. Data description

The HSCM is a university hospital affiliated with the University of Montreal and belonging to one of the five integrated university health and social services centers (CIUSSS) in the city. The hospital can accommodate up to 62,000 patients per year. A total of 35 physicians worked in this hospital in 2017. The demand of the acute care area is covered by six 8-h length shifts starting at 7am, 8am, 3pm, 4pm, 11pm, and 12am. The demand of the fast-track clinic is covered by four 8-h length shifts starting at 7am, 8am, 3pm, and 4pm. Day shifts (S_D) start at 7am and 8am, evening shifts (S_E) start at 3pm and 4pm, and night shifts (S_N) start at 11pm and 12am. This dis- tribution of shifts means that the night acute care physi- cians assume patients of the fast-track clinic.

Anonymous data were collected from March 2008 to September 2017, including approximately 600,000 entries. Each entry represents a physician consultation.

Variables are divided into two groups. The first group corresponds to the patients' characteristics and the second group to the consultation characteristics. The variables used in the study are presented in Table 1. The historical information contains 36,021 differ- ent worked shifts, representing 288,168 h of work. This information indicates that 45.21% of patients are women and that 54.79% are men. The average age of patients is 51.75 years, 46.66% of patients were treated in the acute care area and 0.27% of patients had a language barrier (i.e., the patient did not speak French or English), and 84% of the records of the historical information correspond to patients who saw only one physician (sometimes a patient is seen by more than one doctor). The proportion of patients in each triage level is presented in Table 2. The lower the level is, the more urgent the patient case is.

3.2. Demand forecasting

The ability to accurately forecast the demand repre- sents an important (and probably one of the first) step for developing robust decision support tools in scheduling and in resource planning for health care in general. In fact, the incorporation of accurate demand forecasts beside other factors including bed availability, laboratory testing, and nursing

Table 1. Types of variables collected.

Patients variables	Consultation variables
Age	Physician code
Gender	Consultation date
Language barrier	Type of emergency
Arrival date	Kardex[a] codes
Arrival type	Examen codes
Departure date	Complaints
Departure type	Length of consultation
Triage level[b]	

[a]Quebec Nursing Documentation Tool. [b]Triage level determined by the Canadian Triage and Acuity Scale (CTAS).

availability in the different planning levels, i.e., stra- tegic, tactical and operational, could help to improve several operational issues such as physician over- utilisation and long patient waiting times.

To estimate the total number of patients b_{edi} arriving to the emergency room during day $d \in D$ at time interval $i \in I_d$ for emergency type $e \in E$ we used a two-step methodology. First, we forecast the total number of patients b_{ed} arriving at day $d \in D$ for emergency type $e \in E$ (temporal aggregation). This temporal aggregation of the demand was then dis- tributed among the time periods of each day by means of an *intra-day distribution model*.

The total number of patients b_{ed} was estimated by using a *decomposable time series model* (Harvey & Peters, 1990) with three main model components: *growth, seasonality*, and *holidays*. These components (presented in Equation (1)) represent the *growth* function (g_{ed}) which models *non-periodic changes* in the value of the time series, the *periodic changes* function (s_{ed}) modelling weekly or yearly seasonality, and the *effects of holidays* function (h_{ed}) including effects from holidays, such as Easter and Christmas day. The error term ϵ_{ed} represents irregu- lar changes in demand, which are not accommodated by the time series model.

$$b_{ed} = g_{ed} + s_{ed} + h_{ed} + \epsilon_{ed}, \forall e \in E \quad (1)$$

Equation (1) is estimated with Facebook Prophet (Taylor & Letham, 2018). This tool uses an *additive regression model* with four components: i) a piecewise linear or logistic growth curve to detect changes in trends by selecting change points from the historical data; ii) a yearly seasonal component modelled using Fourier series; iii) a weekly seasonal component using dummy variables; iv) a user-provided list of relevant holidays. The reader is referred to (Taylor & Letham, 2018) for more information on how Facebook Prophet works.

Let r_{edi} be a parameter denoting the mean of the percentage of the total number of patient arrivals during day $d \in D$ for emergency type $e \in E$ that are allocated to time period $i \in I_d$. This parameter is estimated with nine-year historical data providing information on the number of patients arriving for each emergency type at each time interval of the day. As we assume the month of the year does not have a significant effect in the intra-day distribution model, r_{edi} only varies according to weekdays or weekends. The aggregate estimated demand $\hat{b}_{ed}$ is distributed among the periods of the day by using r_{edi}. Hence, the estimate of the number of patients of type $e \in E$ arriving at time period $i \in I_d$ of day $d \in D$ ($\hat{b}_{edi}$) is given by:

Table 2. Proportion of patients in each triage level.

Triage level	Percentage
1	1.67%
2	23.24%
3	47.78%
4	21.70%
5	5.61%

$$\hat{b}_{edi} = \lfloor r_{edi} \cdot \hat{b}_{ed} \rceil, \forall e \in E, d \in D, i \in I_d, \quad (2)$$

where $\lfloor \cdot \rceil$ represents the nearest integer value.

3.3. Physicians′ productivity

The goal of the productivity index is to create a fair indicator that will reflect the capacity of each physi- cian to serve new patients (i.e., patients who were not handed over by another physician). These patients represent 84% of the total time of consults. Hence, treating new patients defines the main task of ED physicians. The HSCM currently allocates patients to physicians based on the acuity level which is determined by the triage level and which does not reflect the capacity to treat new patients, since it is not correlated with consultation length (Yoon, Steiner, & Reinhardt, 2003). That is why we propose computing the productivity index by taking into account two components: an estimation of the num- ber of patients each physician serves at each time interval of his shift, and the patients’ “heaviness” (i.e., the consultation length).

The patient heaviness is computed for each patient based on historical data. To do so, we divided the consultation length into three categories: c_1 for con- sultations with a length between 0 and 15 min, c_2 for consultations with a length between 15 and 30 min and c_3 for consultations with a length larger than 30 min. Table 3 presents the proportion of patients observed in each category. The most represented category is c_3 including around 50% of the patients. Categories c_1 and c_2 contain 20.87% and 30.01% of the patients, respectively. The consultation length is defined as the duration between the moment the patient enters the physicianĂ’Źs room and the moment he leaves. The consultation time is rounded to the nearest minute. Note that this time is only computed for the first consultation, i.e., if a patient sees another physician later, this latter consult will not be considered.

Each category owns a mean consultation length denoted by t_1, t_2 and t_3 as shown in Table 3. Let $\bar{t}$ be the mean consultation length for all the consulta- tions (equal to 36min). With these two values, we

Table 3. Proportion of patients in each category of length consultation with their associated weight.

Category	Percentage	Mean consultation length (t)	Weight ($\hat{w}$)
c_1	20.87%	10 min	0.3
c_2	30.01%	22 min	0.6
c_3	49.12%	55 min	1.5

then affect an estimated weight to each patient $\hat{w}$ given by:

$$\hat{w} = \frac{t}{\bar{t}}, \forall t \in \{t_1, t_2, t_3\} \quad (3)$$

We construct an intermediate productivity index P_{ps} based on the number of patients (counted with their weight $\hat{w}$) treated by the physician $p \in P$ during each shift $s \in S$ observed in the historical data. This pre- liminary statistical study determined that there is not a significant effect on the month of the year in this index. However, this index varies according to the day, the type of shift and the type of emergency. Let $\hat{p}_{pjs}$ be the estimate of the productivity index for each physician $p \in P$ during day $j \in J$ and shift $s \in S$. Set J denotes the days of week. This productivity index $\hat{p}_{pjs}$ is given by:

$$\hat{p}_{pjs} = \frac{\sum_{s'=1}^{N_{pds}} P_{ps'}}{N_{pds} \times l_s}, \forall p \in P, j \in J, s \in S \quad (4)$$

$$P_{ps'} N_{pds} \times l_s, \forall p \in P, j \in J, s \in S \quad (4)$$

Where l_s denotes the length of the shift $s \in S$ and N_{pds} is the total number of shifts $s \in S$ worked during day $j \in J$ by physician $p \in P$. This productivity index is not biased by the variation of productivity through- out a week and precisely reflects the capacity of each physician to consult new patients during one time period of the day.

3.4. Mathematical model

In this section, we describe the mathematical model for the productivity-based physician scheduling pro- blem. We developed a mixed-integer programming model that generates a near-optimal schedule responding to different objectives. The objective of the proposed model is to **reduce unmet patient demand** by matching capacity (physicians’ produc- tivity) with demand (number of patients arriving per hour). Since physician retention is one of the most critical issues facing hospital administrations (Carter & Lapierre, 2001), the model also aims to **minimise physicians’ dissatisfaction and unfairness** in the distribution of shifts between physicians. Definition of satisfaction may vary from one hospital to the other and from one physician to the other. However, in each hospital, there is a general agree- ment on what should be considered. As requested by the person in charge of the schedule planning at HSC, physicians’ dissatisfaction is measured as the sum of differences between the number of shifts wanted and the number of shifts allocated. Unfairness between physicians is measured as the sum of differences between the number of day shifts and the number of evening and night shifts allocated to the schedules. Note that to consider individual preferences, we exclude the physicians to which a measure does not apply from the calculation. Physician schedules in EDs are often subject to various constraints. These constraints are divided into four categories (Beaulieu et al., 2000): *compulsory constraints, ergonomic con- straints, distribution constraints* and *goal constraints*. Compulsory constraints are based on rules that must be absolutely enforced, such as, a rest of 16 h between two consecutive shifts, respecting physicians' avail- abilities, and the consideration of physicians' qualifi- cations to perform certain shifts. For instance, physicians over 50 years-old may not be allocated night shifts. The largest number of constraints are grouped into the category of ergonomic constraints. These constraints aim to improve the quality of the schedules produced by limiting the number of suc- cessive working days belonging to ergonomic con- straints and by enforcing a certain continuity in shifts during the weekend. Distribution constraints limit the number of certain types of shifts allocated to schedules. For instance, each schedule must con- tain a maximum number of night and weekend shifts allocated within the planning horizon. Finally, goal constraints are based on rules which cannot always be satisfied. For instance, the under-covering and over- covering of patient demand, the under-staffing and over-staffing of the quantity of physicians required to work in each type of shift, and the respect of physi- cians' preferences. The reader is referred to 6 for the definition of the decision variables and parameters, as well as the mathematical formulation for the produc- tivity-based physician scheduling problem in EDs.

4. Numerical results

This section presents the computational results obtained after testing the proposed model on real- world-based data. The goal of our study is to improve ED scheduling in order to minimise patient wait times. We want to show that the incorporation of physicians' productivity is an efficient way to better schedule ED physicians without additional costs. First, we describe the generation of the different instances. Then, we eval- uate the differences in schedule quality when using different values for the penalties in the objective func- tion. Finally, we discuss the results.

4.1. Instances generation

To evaluate the ability of our model to cope with demand and productivity variation, we use four sce- narios with different weights (p_1,p_2,p_3 and p_4) asso- ciated with each criteria in the objective function. This allows us to find the configuration that better meets the objectives set by HSCM. We remark that the minimisation of under-staffing is the most impor- tant objective for HSCM. Hence, this objective is associated with the largest weight in the objective

function. Scenario 1 reproduces the current situation of the HSCM ED when the coordination between physicians' productivity and patient demand is not optimised. Scenario 2 characterises the situation when all the goals of the objective function are fairly taken into account. Scenario 3 (resp. Scenario 4) represents the situation when the coordination between physicians' productivity and patient demand (resp. physicians' dissatisfactions and unfairness) is prioritised. The different characteristics of the scenar- ios are summarised in Table 4.

4.2. Accuracy of the demand forecast model and Physicians′ productivity index

In Section 3.2 we introduce a method to forecast the hourly patient demand using Facebook Prophet (Taylor & Letham, 2018). This method takes into account the potential seasonality of the patients' demand and the effects of holidays. Figure 1 shows there is an upward trend for the patients' demand in fast-track clinic. It also shows that Christmas and New Year's day create a punctual impact on the patients' demand. A weekly trend is also determined in both fast-track clinic and acute care area.

Figure 2 shows how the forecast demand behaves on its own, compared to the real demand for the first week. This first week represents how a weekly trend is observed in both acute care area and fast-track clinic. We remark that the forecast demand doesn't exactly predict the sudden variations into 1 day, but only provides the daily trend.

In Section 3.3 we introduced a method to compute the productivity ratio of each physician. These productivity ratios present two particular features which, to the best of our knowledge, have not yet been introduced in the literature. First, we assume that the physicians' productivity varies from one physician to another. Indeed, as shown in Figure 3, the distribution of the individual mean productivity ratio in both acute care area and fast-track clinic significantly varies among all physicians. Second, we assume that the physicians' productivity varies within the days of the week.

In Figure 4, we present the weekly variation of the average of the estimate *Productivity ratio* but also the weekly variation of the average of the physicians' productivity. The *Productivity ratio* fits with this productivity in both acute care area and fast-track clinic that proves the relevance of the *Productivity ratio*. In

Table 4. Scenarios.

Scenarios	S_1	S_2	S_3	S_4
Weight for under-staffing (p_1)	100	100	100	100
Weight for over-covering and under-covering (p_2)	0	1	10	1
Weight for physicians dissatisfactions (p_3)	10	1	1	10
Weight for unfairness (p_4)	10	1	1	10

We use a planning horizon of 13 weeks as in HSCM.

acute care area, the productivity of physicians during the night shift is, on average, higher than the produc- tivity during the day and the evening shifts. In gen- eral, physicians' productivity is higher during the weekend than during a weekday for both acute care area and fast-track clinic. The productivity in acute care area is not significantly affected by the weekdays. However, we observed a significant difference in the

Figure 1. Seasonality of patientsT demand.

productivity of physicians for the fast-track clinic during all days.

4.3. Results

This section presents the results of our study. First, we compare the results of the four scenarios by analysing, from a broad perspective, the values for the different

Figure 2. Comparison of patientsT arrival in the acute care area and in the fast-track clinic per day of the first week.

objectives. Then we analyse, in detail, the distribution of the differences between patients' demand and physicians' productivity throughout the planning horizon.

The computational experiments were performed on a Mac OS X operating system, 16 GB of RAM and 1 processor Intel Core i7 running at 2.5 GHz. The algorithm to solve the problem was implemented in the Julia programming language (JOC, 2015). The instances were solved with CPLEX version 12.8.0.0. A relative gap tolerance of 5% was set as a stopping criterion for solving the MILP with CPLEX.

We define seven categories of indicators to determine the performance of each scenario. The first category measures the total under-staffing. Categories two and three measure the under-covering of patients' demand in acute care area and fast-track clinic, respectively. Categories four and five measure the over- covering of patients' demand in acute care area and fast- track clinic, respectively. The sixth category measures the fairness among physicians and, finally, the seventh category denotes the performance in respecting physi- cians' preferences. Results in Table 5 (part 6) does not include the physicians with individual preferences on night shifts. Table 5 presents the results of each category for each scenario.

Scenario 1 reproduces the current schedule of the HSCM ED. In this case, even though the coordination between physicians' productivity and patients' demand is not optimised, the fairness in the distribu- tion of shifts and physicians' preferences is optimised. While the values for fairness and for physicians' pre- ferences are the best among all scenarios, the values related to the under-covering and over-covering of patients' demand are, indeed, the worst among all scenarios. As a result, we analysed the results of the three last scenarios in order to improve under- covering and over-covering of demand without degrading the number of under-staffed shifts, the fair- ness between physicians, and the respect of physicians' preferences.

4.4. Total under-staffing

All three scenarios (S_2, S_3 and S_4) present the same values for under-staffing, as the large weight given to p_1 ensures minimising under-staffing as much as possible.

4.5. Under-covering and over-covering of patients' demand

Scenarios 2 and 3 present similar results regarding the under-covering and over-covering of patients' demand. The percentage of uncovered demand in Scenario 4 is reduced when compared to Scenario 1.

Figure 3. Distributions of physicians' productivity into the type of emergencies.

Figure 4. Weekly variations of physicians' productivity.

Table 5. Computational results of each scenario.

Scenarios	S_1	S_2	S_3	S_4
Objective value	960	2698	24,986	3674
Gap (%)	2.08	4.52	4.93	4.98
CPU Time (sec)	75	10,774	2779	9458
1. Under-staffing				
Number of under-staffed shifts	0	0	0	0
Number of allocated shifts	1014	1014	1014	1014
2. Under-covering of patients' demand in acute care area				
Total uncovered demand	1125	434	387	526
Mean(std) uncovered demand per hour	0.93(0.64)	0.51(0.39)	0.47(0.37)	0.59(0.45)
Maximum uncovered demand per hour	3.7	2.05	2.13	2.24
% uncovered demand	18.95	7.3	6.52	8.85
3. Under-covering of patients' demand in fast-track clinic				
Total uncovered demand	1631	790	717	874
Mean(std) uncovered demand per hour	1.43(1.18)	0.77(0.76)	0.76(0.70)	0.81(0.74)
Maximum uncovered demand per hour	7	4.01	2.36	3.97
% uncovered demand	24.98	12.1	10.99	13.38
4. Over-covering of patients' demand in acute care area				
Total over-covered demand	199	240	260	251
Mean(std) over-covered demand per hour	0.50(0.46)	0.35(0.43)	0.36(0.42)	0.38(0.39)
Maximum over-covered demand per hour	2.28	2.2	1.99	1.99
5. Over-covering of patients' demand in fast-track clinic				
Total over-covered demand	1412	879	934	848
Mean(std) over-covered demand per hour	1.36(0.98)	0.76(0.52)	0.75(0.53)	0.77(0.54)
Maximum over-covered demand per hour	4.89	2.42	3.71	2.52
6. Difference between number of day shifts and the number of evening and night shifts				
Mean(std)	0.32(0.47)	1.41(5.63)	9.29(11.21)	0.11(0.32)
Minimum per physician	0	0	0	0
Maximum per physician	1	33	43	1
Total number of shifts creating an unbalance	11	48	316	4
7. Difference between number of wanted shifts and the number of allocated shifts				
Mean(std)	2.71(7.53)	4.79(9.16)	7.38(10.70)	2.91(7.46)
Minimum per physician	0	0	0	0
Maximum per physician	33	34	41	33
Total number of allocated shifts not wanted	92	163	251	99

However, this percentage is slightly larger than the one obtained for Scenarios 2 and 3.

4.6. Difference between the number of day shifts and the number of evening and night shifts

The difference between the number of day shifts and the number of evening and night shifts allocated to physicians gives an idea of the fairness in the distribution of shifts between them. Computational results show that schedules are more fair when Scenarios 1 and 4 are used to solve the problem, rather than using Scenarios 2 and 3.

4.7. Difference between the number of wanted shifts and the number of allocated shifts

The differences between the number of wanted shifts and the number of allocated shifts (corresponding to the number of unwanted shifts) denote the respect of physicians' preferences. Scenario 4 appears to be the best option for the number of unwanted allocated shifts. On the contrary, Scenarios 2 and 3 are simply unacceptable as the number of unwanted shifts

Table 6. Improvement of the objectives functions. doubles.

Objectives Improvement Scenario 4 seems to be the scenario that responds

Number of under-staffed shifts 0% best to the objectives defined. Results related to under-covering and over-covering of patients'

% uncovered demand in acute care area % uncovered demand in fast-track clinic Number of shifts creating an unbalance 10.10% 11.60% 36.36% demand for Scenario 4 are close to the ones for

Number of allocated shifts not wanted −7.61%

Scenarios 2 and 3, where the coordination between productivity and demand is best. Results concerning fairness between physicians are close to the ones in Scenario 1, where only fairness and the respect of physicians' preferences are optimised. They are also significantly better than those for Scenarios 3 and 4, which are not acceptable as the degradation of these objectives is simply too large. While the results related to the physicians' preferences are worse than for those in Scenario 1, they are acceptable nonethe- less. We summarise in Table 6 the improvement in the objectives when Scenario 4 is used instead of Scenario 1 (i.e., the current situation). Negative improvements correspond to a degradation in the corresponding objective.

From Table 6 we observe that the improvement in the alignment between productivity and demand, by using Scenario 4, is significant when compared to using Scenario 1. The percentage of uncovered demand is reduced by 10.10% and by 11.60% in

5. acute care area and fast-track clinic, respectively.

Conclusions While the unfairness in the allocation of shifts decreases by 36.36%, unfortunately, the number of unwanted allocated shifts increases by 7.61%. The graphical results of this comparison are presented in Figure 5.

Figure 5 shows how the physicians' productivity (corresponding to the number of patients the phy- sician can treat) behaves, on average, in acute care area and fast-track clinic under Scenario 1 (i.e., current schedule) and under Scenario 4 (i.e., improved schedule). The productivity of the

This study shows a substantial gap between the cur- rent schedules used at HSC and the timing of patient arrivals. Incorporating physicians' productivity in the design and allocation of physician schedules allows to better align the patients demand to the availability of physicians. It also offers a cost-free way (i.e., it is not necessary to hire more physicians) to decrease some of the patients' unmet demands and waiting times without significantly degrading fairness and physi- cians' preferences.

improved schedule offers better alignment with

We proposed a forecasting model to predict the patient arrival rates when compared to the pro-

arrival of patients and a productivity index including ductivity of the current schedule. This improve-

patients' heaviness (i.e., consultation time), while also ment is particularly visible on the weekend in the

taking into account the productivity variations within fast-track clinic. Indeed, in the current schedule,

a week. These parameters are incorporated into an the physicians' productivity does not follow the optimisation model that seeks to improve the coordina- patients' demand variations. When the demand is tion between physicians' productivity and patient arri- high in the morning physicians' productivity is vals while including physicians' preferences, fairness lower than the afternoon, when the demand is between the allocation of schedules for different physi- lower.
cians, and common work rules in the composition of Figure 6 shows the results of the alignment schedules. The proposed model can be generalised to between physicians' productivity and patients' other EDs: the constraints included in the mathematical demands in acute care area and fast-track clinic for formulation are common and relevant to most EDs and the first week. Apart from showing that patients' the objective function can be easily adapted to fit differ- demands are subject to high variations, this figure ent fairness and satisfaction definitions. also shows that the new schedules improve the gap
Next step should be to test one of our schedules between the patients' demand and the ED's capacity in vivo and adjust our productivity measure with to serve new patients.
feedback from the field.

Figure 5. Results of the alignment between physiciansT productivity and patients demand in acute care area and fast-track clinic.

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Appendix

Appendix A. Mathematical model

Appendix A1. Variables and parameters description

Let p $b$covering $^{À}{}_{edi}$ 2 xP be $_{pds}$ is be non-negative allocated and a binary under-covering to variable shift slack s 2 that variables S of in takes demand day denoting value d 2 D. for 1 Let if emergency the physician $b^{þ}{}_{edi}$ over- and type e 2 E tively. Non-negative during time variables period si $^{þ}{}_{p}$ 2 and I_d in day d 2 D, respec- $s^{À}{}_{p}$ represent positive and negative deviations between the number of day shifts and the number of evening and night shifts allocated to physician p 2 P, respectively. Let g_p ^{þ}and g_p ^{À}denote the positive and negative differences between the number of shifts wanted and the number of shifts allocated to physi- cian p 2 P, respectively. Let senting the number of missing $c^{À}{}_{ds}$ physicians be a slack variable repre- required to cover shift s 2 S during day d 2 D. The parameters used in the productivity-driven physician scheduling model are defined in Table A1.

Table A1. List of parameters.

Name Type Definition δ_{si} Binary Takes value 1 if shift s covers time period i, it assumes value 0 otherwise w_d Integer Indicates the day of the week (e.g. Monday,. . ., Sunday) associated with day d $^{\wedge}{}_{ppjs}$ Float Productivity index for physician p, during day of the week j and shift s $^{\wedge}{}_{bedi}$ Integer Estimated number of patients arriving at time period i in day d for emergency type e w_p Integer Number of shifts wanted by physician p c_{ds} Integer Number of budgeted physicians to cover shift

s during day d e_{ps} Binary Takes value 1 if physician p is qualified to work

on shift s, it assumes value 0 otherwise d_{pds} Binary Takes value 1 if physician p is available to work

on shift s of day d, it assumes value 0 otherwise $s^{max}{}_{p}$ Integer Maximal number of shifts preferred by physician

p $n^{max}{}_{p}$ Integer Maximal number of night shifts preferred by physician p n^w Integer Maximal number of weekends that can be

allocated to each physician p_1, p_2, p_3, p_4 Integers Weight of each objective in the objective function

Appendix A2. Mathematical formulation

The multi-objective function of the model (5) mini- mises under-staffing, under-covering, and over- covering of the patients' demand. This objective func- tion also ensures a certain fairness, minimising the differences between the number of day shifts and the number of evening and night shifts allocated to each physician. Objective (5) also ensures the satisfaction of physicians' preferences, minimising the difference between the number of shifts wanted and the number of shifts allocated.

$$\min p_1 \sum_{d \in D} \sum_{s \in S} c^-_{ds} + p_2 \sum_{e \in E} \sum_{d \in D} \frac{1}{2}\left(b^+_{edi} + b^-_{edi}\right) + p_3 \sum_{p \in P} \frac{1}{2}\left(s^+_p + s^-_p\right) + p_4 \sum_{p \in P} \left(g^+_p + g^-_p\right) \quad \text{(A.1)}$$

The productivity-driven physician scheduling model is subject to the following constraints:

Compulsory constraints: Constraints (A.2) guarantee that each physician is allo- cated a maximum of one shift per day.

$$\sum_{s \in S} x_{pds} \leq 1, \forall p \in P, d \in D \quad \text{(A.2)}$$

Constraints (A.3) and (A.4) ensure a minimum rest time of 16 h between two consecutive shifts.

$$\sum_{s \in S_E \cup S_N} x_{pds} + \sum_{s \in S_D} x_{p(d+1)s} \leq 1, \forall p \in P, d \in D \setminus \{|D|\} \quad \text{(A.3)}$$

$$\sum_{s \in S_N} x_{pds} + \sum_{s \in S_D \cup S_E} x_{p(d+1)s} \leq 1, \forall p \in P, d \in D \setminus \{|D|\} \quad \text{(A.4)}$$

Constraints (A.5) ensure that the availabilities for each physician are respected.

$$x_{pds} \leq d_{pds}, \forall p \in P, d \in D, s \in S \quad \text{(A.5)}$$

Constraints (A.6) ensure that physician qualifications to perform certain shifts are respected.

$$x_{pds} \leq e_{ps}, \forall p \in P, d \in D, s \in S \quad \text{(A.6)}$$

Ergonomic constraints:

Constraints (A.7) ensure that after a physician ends a night shift at day $d + 1$ (which means the physician starts this shift the day d), this physician is not allowed to work a night shift the day after.

$$\sum_{s \in S_N} \left(x_{pds} - x_{p(d+1)s} + x_{p(d+2)s}\right) \leq 1, \quad \forall p \in P, d \in D \mid d \leq |D| - 2$$

(A.7)

Constraints (A.7) ensure that after a physician ends a night shift at day d þ 1 (which means the physician starts his shift the day d), the physician is not allowed to work a day or an evening shift the day after.

$\sum_{s2S_N}$

x_{pds} þ $\sum$

$s2S_D[S_E$

$x_{p\eth d þ 2 Þ s}$ 1, ∀ p 2 P, d 2 Djd jDj À 2

(A.8)

Notably, constraints (A.7) and (A.8) ensure together that a day-off is allocated after a sequence of night shifts.

Constraints (A.9) prevent isolated nights at each physi- cian schedule. $\sum_{s2S_N}$ðx_{pds} À $x_{p\eth d þ 1 Þ s}$ þ $x_{p\eth d þ 2 Þ s}$Þ ! 0,

∀ p 2 P, d 2 Djd jDj À 2

(A.9)

$\sum_{i2I_d}$

In the context of HSCM ED, these constraints are always respected. However, a minimisation of the number of iso- lated nights is sometimes preferred. In this case, let n_{pd} be a binary variable denoting whether the night of day d 2 Dnfj$D$jg is an isolated night for the physician p 2 P. Thus, constraints (A.9) are replaced by constraints (A.10) and the term denoted by equation (A.11) is added in the objective function (A.1) where p_5 is the weight of the objective. $\sum_{s2S_N}$ðx_{pds} À $x_{p\eth d þ 1 Þ s}$ þ

$x_{p\eth d þ 2 Þ s}$Þ!À$n_{p\eth d þ 1 Þ}$, ∀ p 2 P, d 2 Djd jDj À 2

(A.10)

p_5

$\sum_{p2P}$

$\sum_{d2D\mathrm{nfj}D\mathrm{jg}} n_{pd}$ (A.11)

Constraints (A.12) set the maximum number of conse- cutive working days that can be allocated to a physician.

$\sum_{s2S, r¼d..dþ3} x_{prs}$ 3, ∀ p 2 P, d 2 Djd jDj À 3 (A.12)

Constraints (A.13) guarantee that each physician is allo- cated a maximum of five shifts within seven consecutive days.

$\sum_{s2S, r¼d..dþ6} x_{prs}$ 5, ∀ p 2 P, d 2 Djd jDj À 6 (A.13)

Constraints (A.14) guarantee that each physician is allocated a maximum of 3-night shifts within seven consecutive days.

$\sum_{s2S_N, r¼d..dþ6} x_{prs}$ 3, ∀ p 2 P, d 2 Djd jDj À 6 (A.14)

Constraints (A.15) ensure that schedules do not contain two consecutive fast-track clinic shifts.

x_{pds} þ $x_{p\eth d þ 1 Þ s}$ 1, ∀ 2 P, d 2 Dnfj$D$jg, s 2 S^F (A.15)

Constraints (A.16), (A.17), and (A.18) ensure a certain con- tinuity within weekend shifts worked by each physician. We denote by D_F and Ds the subsets of days D including Fridays and Sundays, respectively. If a physician works a day shift on Saturday, a day shift is also worked on Sunday by the same physician. If a physician works an evening shift on Friday, evening shifts are also worked on Saturday and Sunday by the same physician. If a physician works a night shift on Friday, night shifts are also worked on Saturday and Sunday by the same physician.

$\sum_{s2S_D}$

x_{pds} ¼ $\sum_{s2S_D}$

$x_{p\eth d þ 1 Þ s}$, ∀ p 2 P, d 2 Dsjd<jDj (A.16)

$\sum_{s2S_E}$

$$x_{pds} = \sum_{s \in S_E} x_{p(d+1)s}, \forall p \in P, d \in D_F \cup D_S | d < |D| \quad (A.17)$$

$$\sum_{s \in S_N} x_{pds} = \sum_{s \in S_N} x_{p(d+1)s}, \forall p \in P, d \in D_F \cup D_S | d < |D| \quad (A.18)$$

Constraints (A.19) forbid the allocation of two consecutive weekends to each physician schedule.

$$(x\sum_{p(d+7)s} + x_{p(d+8)s}) \; 2 \cdot (1 - \sum_{s \in S} \; x_{pds}), \forall p \in P, d \in D_S | d \; \{|D| - 8 \quad (A.19)$$

Distribution constraints:

Constraints (A.20) and (A.21) guarantee that each phy- sician does not work more than a maximum desired num- ber of shifts and a maximum desired number of night shifts, respectively.

$$\sum_{s \in S, d \in D} x_{pds} \; s^{\max}_{p}, \forall p \in P \quad (A.20)$$

$$\sum_{s \in S_N, d \in D} x_{pds} \; n^{\max}_{p}, \forall p \in P \quad (A.21)$$

Constraints (A.22) guarantee that a physician does not work more than n^w weekends during the time horizon.

$$\sum_{s \in S, d \in D_S} x_{pds} \; n^w, \forall p \in P \quad (A.22)$$

Goal constraints:

Constraints (A.23) ensure that the number of physicians working during day d at shift s is lower than or equal to c_{ds}.

$$\sum_{p \in P} x_{pds} + c^{-}_{ds} \; c_{ds}, \forall d \in D, s \in S \quad (A.23)$$

Constraints (A.24) ensure that the number of patients treated by physicians during day and evening shifts is equal to the patients' demand subject to some adjustments related to under-covering and over-covering.

$$\sum_{s \in S^e_D \cup S^e_E} \sum_{p \in P} \delta_{si} \cdot \hat{p}_{pw_ds} \cdot x_{pds} + b^{-}_{edi} - b^{+}_{edi} = \hat{b}_{edi}, \forall d \in D, i \in I_d, e \in E \quad (A.24)$$

Since there are no physicians allocated to fast-track clinic during night shifts, constraints (A.25) ensure that the number of patients treated by physicians of acute care area in the night shifts is equal to the total patients' demand in both acute care area and fast-track clinic, subject to some adjustments related to under-covering and over-covering.

$$\sum$$

s 2 $S^A{}_N$

$\sum_{p2P}$

$^{\delta}{}_{si}$ Á ^$p_{pw_d s}$ Á x_{pds} þ $b^{\text{À}}{}_{Adi}$ À $b^{\text{þ}}{}_{Adi}$

¼ $\sum_{e2E}$

(A.25)

Constraints (A.26) guarantee that each physician works approximately the same number of day shifts as evening and night shifts.

$\sum_{d2D}$

^b_{edi}, ∀ d 2 D, i 2 I_d

$\sum_{s2S_D}$

x_{pds} þ $s^{\text{À}}{}_p$ À $s^{\text{þ}}{}_p$ ¼ $\sum_{d2D}$

$\sum$

$s2S_E[S_N$

x_{pds}, ∀ p 2 P (A.26)

Constraints (A.27) guarantee that each physician works approximately the number of shifts wanted.

$\sum_{d2D}$

$\sum_{s2S}$

x_{pds} þ $g_p{}^{\text{À}}$À $g_p{}^{\text{þ}}$¼ w_p, ∀ p 2 P (A.27)